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UNIVERSITY OF ALBERTA

SUPERCOMPOSITE MODELS AND RARE PROCESSES

BY

KENNETH ANDREW PETERSON

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY.

IN

THEORETICAL PHYSICS

DEPARTMENT OF PHYSICS

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FALL 1991



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K. Andrew Peterson

Permanent Address:

1209 Wiggins Avenue,

Saskatoon, Saskatchewan, Canada.

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THE UNIVERSITY OF ALBERTA
FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled SUPERCOMPOSITE MODELS AND RARE PROCESSES submitted by Kenneth Andrew Peterson in partial fulfilment of the requirements for the degree of Doctor of Philosophy. in Theoretical Physics.

Dr. B.A. Campbell

...*B.A. Campbell*...

Supervisor

Dr. F.C. Khanna

...*F.C. Khanna*...

Dr. G. Ludwig

...*G. Ludwig*...

Dr. L.G. Greeniaus

...*L.G. Greeniaus*...

Dr. P.J. Watson

...*P.J. Watson*...

External Examiner

Dr. A.N. Kamal

...*A.N. Kamal*...

Date...*Sept 25, 1991*...

To Dawn:

For bearing with me through the final stages of my Ph.D.

This is for both of us.

To my parents:

For instilling in me the quest for knowledge
and the encouragement to be what I wanted to be.

To all the teachers I have had
during my nearly quarter century life as a student:

May I never cease to learn.

ABSTRACT

This thesis will deal mainly with induced contact interactions in composite and supercomposite models that might contribute to various rare processes, most notably flavour-changing interactions. By supercomposite, we refer to composite models that have an underlying supersymmetry. Whereas a purely composite model can have induced four-fermion dimension-six operators suppressed by the square of the composite mass scale, supercomposite models can have induced dimension-five two-scalar, two-fermion induced interactions suppressed by only one power of the composite mass scale. The two-scalars would be the supersymmetric scalar partners of the fermions, and therefore this interaction could be "dressed" to a four-fermion interaction through the exchange of a gaugino (the supersymmetric partners of gauge bosons). This would lead to effective four-fermion interactions suppressed by one power each of the composite and supersymmetric mass scales. If the supersymmetric mass scale is significantly lower than the composite mass scale, this can lead to constraints on the composite mass scale that are much more restrictive than constraints on pure composite models.

We investigate the processes, π_{21} decay, muon conversion ($\mu N \rightarrow e N$), the K_L - K_S mass difference (and B - $\bar{B}$ mixing), and the neutron electric dipole moment. For each of these processes we first set limits on the square of the composite mass scale from dimension-six operators induced by a pure composite model. We then set limits on the product of the composite and supersymmetric mass scales from possible supercomposite interactions.

Also in this thesis we investigate possible contributions to the decays $b \rightarrow s \gamma$ and $b \rightarrow s \mu^+ \mu^-$ by an anomalous W quadrupole moment. Such an anomalous quadrupole moment is often suggested in composite gauge boson theories. By using the experiment limits on the decay $B \rightarrow K^* \gamma$, we can constrain the coupling constant associated with this quadrupole moment to, $|\lambda_\gamma| \leq 10$ for a top mass of 135 GeV. Further improvements on this limit could constrain the value to be $|\lambda_\gamma| \leq O(1)$.

PREFACE

Throughout the course of my graduate studies I have been asked on numerous occasions about what I am working on. I invariably answer that I am studying theoretical particle physics. If pressed to elaborate, I would inevitably stall. At most, I would respond that I was working on the physics of such "elementary" particles as quarks and electrons. Sometimes this would satisfy the inquisitor, but more often than not it would just leave them confused. Obviously I have not explained my studies completely to the layperson. Without the ability to explain my studies to the people that I need to reach the most, I feel that my education would be incomplete.

So what do I study and what have I done? My thesis is in the field of theoretical particle physics. Particle physics is the physics of elementary particles and the forces they interact with.

What is an elementary particle? One of the fundamental aspects of science has been to investigate just what the matter we see in front of us is composed of. What we wish to do is subdivide it into its constituent parts. By doing this, we find that matter is made up of molecules which can be further subdivided into atoms. But is this the end? *Atom* is derived from the Greek word *atomos*, meaning indivisible. It has been determined that the atom is not an indivisible object, and the study of its structure has been the subject of nuclear and particle physics in the twentieth century.

Sub-atomic physics actually had its beginning in the late nineteenth century. In 1896, Becquerel discovered radioactivity, a process that Madame Curie later demonstrated to be an atomic process. Also during that time period came the discovery of the electron by Thomson in 1897. Thomson then proceeded to suggest a model of the

atom that involved a "pudding-like" distribution of positive charge with electrons being distributed as "plums" in the pudding. However, Rutherford's famous 1911 scattering experiment challenged Thomson's conclusions. If Thomson's model was correct, then energetic particles shot at atoms would be expected to pass right through. Instead, Rutherford found his particles were being rebounded backwards on occasion. This has often been described as being like shooting a cannon ball at tissue paper and expecting it to come back. Rutherford deduced that, instead of a pudding-like distribution of positive charge, the positive charge must be located in a small, hard and dense nucleus. This view led to Bohr's model of the atom which involved a small hard nucleus of positive charge surrounded by orbiting electrons. The idea that the nucleus was populated by positive protons and neutral neutrons followed. Matter was believed to be composed of elementary electrons, protons and neutrons. However, this model was not to last.

Beginning in the 1930's with the discovery of the muon and lasting well into the 1960's, many new "elementary" particles were discovered. In particular, there was a multitude of "baryons" and "mesons" that were found. In order to clarify this situation, Gell-Mann and Zweig independently suggested that all the known baryons could be produced by combinations of three flavours of "quarks" combined in a threesome. Mesons on the other hand would be quark/anti-quark combinations. For instance, the proton would be composed of two "up quarks" and one "down quark", while the neutron would be composed of one "up quark" and two "down quarks". The third quark, the strange quark was involved with more exotic or "strange" matter. Experimental evidence of quarks came from SLAC in the late 1960's, an accomplishment for which the participants were awarded the Nobel prize for last year (1990). Later, two other flavours of quarks, the charm quark and the bottom quark, were discovered. Interestingly, the charm quark was first suggested by the smallness of a particular process, the mass

difference between a long lived kaon and a short lived kaon (a kaon is one of these "strange" mesons). This was one of the first applications of rare processes to discover new physics, a tradition that this thesis will continue. A sixth quark, the top quark, is widely believed to exist for the same reasons that the charm quark was first suggested. In addition to the six quarks, which are said to come in three families [(up, down), (charm, strange), (top, bottom)], there are also three electron-like particles, the electron, the muon and the tau, along with their associated neutrinos. The electron-like particles and the neutrinos are collectively known as leptons. A fairly general discussion of these matter particles can be found in Section 1.2, with Figure 1.1 being a table of these particles.

The question now becomes: Are these particles truly elementary, or are they some composite structure of some more elementary particle? Perhaps some composite structure would explain why there are three families of quarks or leptons. Perhaps it might even explain their masses. At present, of the energy scales that we are able to probe, these particles appear to be elementary. If they are composites, it is unknown at which probing energy they will exhibit composite traits. This is a question that this thesis seeks to answer, mainly by looking at interactions that would be induced at the composite scale and are small due to the fact that the interactions take place at energy scales far below that scale. Because lower energy interactions would be more prevalent, it is best to look for these composite interactions in processes where low energy effects do not contribute. This is why we look at rare processes.

I previously described particle physics as the study of elementary particles and the forces they interact with. I now turn to these interactions. There are four known forces in nature: gravitation, electromagnetism, the strong force (responsible for holding the nucleus together), and the weak force (responsible for some radioactive decays). Particle physics describes these interactions through the exchange of a particle. The

electromagnetic force is mediated by the exchange of a photon, or quanta of light, while the strong force is mediated by particles known as gluons and the weak force by particles known as W's and Z's. The gravitational force is slightly different, but it too is believed to be mediated by a particle, in this case, the hypothetical graviton. That forces are mediated by particles might at first seem strange, but think of the case of playing catch on a frictionless ice surface with a football. The recoil of throwing the football would set the thrower backward in motion, while absorbing the momentum of the thrown ball would also set the receiver in motion. The thrower and receiver are being put into motion by the throwing and receiving of the ball. In essence, the ball is mediating the force that is separating them. At first glance, this would only appear to work for repulsive forces that do not conserve energy. However, due to the quantum nature of the exchange, the interchanged particles are not "real" particles, but "virtual" ones, allowing for both attractive and repulsive forces that are energy conserving. A list of these interaction particles is to be found in Figure 1.2.

One of the remaining questions that must be asked is how do particles receive mass. It is believed particles become massive because of their interactions with another particle, the Higgs boson. This particle has yet to be discovered and is the subject of much debate. Because of theoretical problems with this particle, many believe it has a composite structure. Others believe it is elementary, the theoretical problems being solved by a new symmetry called supersymmetry. This symmetry requires a partner particle for every known particle; partners that have yet to be found. These particles, if they exist, might be found at some higher mass scale. Once again, their presence could be deduced by looking at how they effect processes that occur at lower energies, an effect that is small owing to their large mass. As before, one would tend to concentrate the search for these effects in processes that had no contributions from the known particles

and interactions.

There are in general three ways of finding new physics when dealing with elementary particles. The first way is to look for new particles. In general, new physics produces new particles with masses equal to the energy scale at which the new physics is relevant. (I equate energy and mass through Einstein's relation $E = mc^2$, where I pick units where the speed of light, c , is equal to one.) The second way is to look for interference between the known interactions and new interactions produced by the new physics. Since these new interactions only become relevant when one approaches the energy scale of new physics, one needs to do experiments at high energies. The same is true for finding new particles. Finally, one could do what I have mentioned above and look for the effects of these new interactions in processes where there is no contribution from the known interactions. One can do this at low energy scales, but at these scales the effects of the high energy contributions will be extremely small. Thus one needs high precision experiments. The searches for rare processes are these high precision experiments.

My thesis, in particular, concentrates on constraints on composite models with supersymmetry that can be obtained from rare processes. In order to produce low mass particles such as quarks and electrons from composite objects with such a large binding energy, it may be desirable to create models that are both composite in nature and also supersymmetric. Once again, one would wish to look for the effect of such theories in rare processes. Here, due to the supersymmetric and composite natures, one has an effect not suppressed solely by the size of the composite mass scale or by the size of the supersymmetric mass scale, but rather through a combination of the two. Since the supersymmetric mass scale is often thought to be far smaller than the composite mass scale, this effect, in general, will be larger than effects that occur in purely composite

theories. It therefore might be easier to see these effects. This thesis seeks to illustrate that if constituent particles of the supposed elementary particles exist, then they must be above a certain energy scale due to the fact their effects have not been detected in certain rare processes. Since supercomposite models have more noticeable effects, this mass scale is higher in those models compared with just a purely composite model.

It is my intention that this preface will enable the reader to gain a greater understanding of the following thesis.

K. Andrew Peterson,
University of Alberta,
Fall, 1991.

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1. INTRODUCTION

1.1 Prologue

The Standard Model of particle physics has been vindicated time after time, yet particle physicists believe it to be incomplete. For this reason numerous theories have been proposed to supplant the Standard Model, many of which fail for the same reasons the Standard Model triumphs. One of these reasons is the Standard Model's ability to accommodate flavour-changing interactions and other rare processes, something that many extensions to the Standard Model predict in excess. This thesis will deal with some of the constraints put on supercomposite and composite models by these rare processes, but before doing that, perhaps a brief history and explanation of the Standard Model, and some of the more common extensions and replacements to the Standard Model, is in order.

1.2 The Standard Model

The development of quantum field theories has been the major influence in our understanding of particle physics. It is a tool to unravel the underlying symmetries of nature that lead to the fundamental forces. The discovery by Einstein in 1905 [Ein.05] of the invariance group of spacetime was the first such underlying symmetry found and led to his theory of gravity. Other symmetries were later introduced that had nothing to do with spacetime. Heisenberg introduced a symmetry called isospin in 1932 [Hei.32], suggesting that the laws governing the strong interactions between protons and neutrons were invariant under isospin. In essence, the proton and neutron could be treated as the same object, a nucleon, as far as the strong interactions went. Other approximate

symmetries such as parity and charge conjugation that were conserved in the strong and electromagnetic interactions followed. Field theory teaches us a very important concept concerning symmetries. Every symmetry associated with a theory bears with it an associated conserved charge [Noe.18]. The best known example of this is the $U(1)$ symmetry of electromagnetism. The conserved charge in that case is the electric charge. Isospin, on the other hand, would be the conserved quantity in an $SU(2)_{\text{isospin}}$ symmetry.

It was initially thought that all these symmetries should be global, however, this is not the case. It has long been known that the $U(1)$ symmetry of electromagnetism is a local symmetry. That is the charged fields have a phase that varies freely from point to point. Indeed, Maxwell's equations can be derived starting only with this local gauge symmetry. In 1954, Yang and Mills [Yan.54] and Shaw [Sha.55] extended this idea of a local gauge symmetry from a $U(1)$ group of phase rotations to non-Abelian symmetries. This led to the prediction of new massless vector bosons in addition to the photon. This was in obvious contradiction to experiment.

In a quantum field theory, forces between elementary particles are generated by the exchange of other elementary particles, the vector bosons mentioned above. By the Heisenberg uncertainty principle, the range of an interaction is given by the inverse of the exchanged particle's mass, a concept established by Yukawa in 1935 [Yuk.35]. For electromagnetism and gravity, whose ranges are infinite, one has a massless photon and graviton. However, the strong and weak forces both have very short effective ranges of 10^{-13} and 10^{-16} cm., corresponding to the mass of the pion and the W respectively. Thus it became necessary to develop theories with approximate symmetries that allowed massive vector bosons. The idea here is to have a spontaneously broken symmetry where the symmetry of the Lagrangian remains unbroken, but the symmetry of the vacuum is broken. Therefore, the symmetry of the physical states need not exhibit the underlying

symmetry of the Lagrangian.

Nambu and Goldstone showed [Nam.61, Gol.61] that, for global symmetries, a spontaneously broken symmetry gives rise to a massless spin-zero Nambu-Goldstone boson. In particular, the breaking of an $SU(2)_L \otimes SU(2)_R$ chiral symmetry down to an $SU(2)_{L+R}$ symmetry gives rise to Nambu-Goldstone bosons that can be identified with the pions. This led to many of the successes of current algebra.

The role of spontaneous symmetry breaking in local gauge theories was worked out by Anderson, Higgs, Brout and Englert, Guralnick, Hagen and Kibble in the 1960's [Hig.64, Kib.67, Gur.64, Eng.64, And.63]. They showed that the massless gauge bosons of the Yang-Mills theory and the massless Nambu-Goldstone bosons could both be circumvented by what is now known as the "Higgs mechanism". In this theory, the massless Nambu-Goldstone bosons are "eaten" by the Yang-Mills vector bosons, giving the vector bosons an additional longitudinal degree of freedom which manifests itself as a mass. 't Hooft [tHo.71] later showed these theories could be renormalized, that is all the infinities of the theory could be absorbed in terms of a finite number of fundamental parameters.

The ground work for the Standard Model is now set. Even before 't Hooft showed the renormalizability of gauge theories with spontaneous symmetry breaking, a unification of the weak and electromagnetic interactions was proposed by Glashow [Gla.61], by Weinberg [Wei.67], and by Salam and Ward [Sal.64, Sal.68]. It employed the Higgs mechanism to break an $SU(2)_L \otimes U(1)_Y$ group down to the unbroken group $U(1)_{em}$ of electromagnetism. This required the introduction of two massive charged vector bosons, the $W^\pm$, responsible for weak decays and a hitherto unknown neutral massive vector boson, the Z^0 , which would be responsible for short range neutral current interactions. The relative strengths of the weak and electromagnetic interactions, along

with the relative masses of the W and Z are given by the Weinberg angle, θ_W . Experiments on ν_e , ν_N and $e d$ interactions soon verified the existence of neutral current interactions and found a value of $\sin^2\theta_W$ of 0.215 ± 0.015 [Per.87]. Because the weak coupling constant is now related to the electromagnetic coupling constant through θ_W , one can obtain a value for the W and thus the Z masses from the Fermi constant, G_F , of weak interactions, $M_W = (37 \text{ GeV}) / \sin\theta_W \approx 78 \text{ GeV}$ and $M_Z \approx 90 \text{ GeV}$. The discovery of both these particles at CERN [UA1.83, UA2.83] in 1983 at energies very close to the predicted values was the final vindication of the theory. Besides the W and Z, the theory also predicts a massive neutral scalar, the Higgs, which has yet to be experimentally discovered. As we will see, this particle is the source of many of the theoretical problems the Standard Model has, and is the motivation for many of the extensions to the Standard Model.

So far we have only dealt with electroweak theory. We have yet to deal with the strong force, but before we do that, perhaps we should talk of the fermionic, or matter content of the Standard Model.

The fermions in the Standard Model can be broken into two groups; particles that do not participate in the strong interaction, the leptons, and particles that do participate in the strong interaction, the quarks. Because they do not participate in the strong force, free leptons exist. They are the electron and the electron neutrino, plus two heavier mass copies of the electron, the muon and tau particles, and their associated neutrinos. The reason for the heavy mass copies of normal matter has been a source of mystery ever since the discovery of the muon in cosmic ray experiments done in the 1930's. Neutrinos were suggested by Wolfgang Pauli in 1930 [Kro.64] as a solution to the problem of apparent energy non-conservation in beta decay. It was found that when the neutron decays to a proton plus an electron, all the available energy is not deposited in the

electron. Pauli suggested that this was due to an unseen particle, the neutrino, also being produced in the decay, carrying away some of the energy. The fact that the neutrino also comes in different flavours was not established until 1961, when experiments on $\nu_\mu p \rightarrow \mu^- n$ were carried out [Dan.62]. To a very high degree of accuracy, the neutrino is massless, stable neutrinos being allowed only a few eV mass for the cosmological reason of not overclosing the Universe. However, some very recent experimental evidence [Sim.85, Sim.89] suggests that an unstable 17 keV neutrino might exist. Massless neutrinos are an inherent assumption of the Standard Model in its simplest form. The neutrino only participates in the weak interaction, and as such requires only a left-handed chiral piece, ν_L . Dirac mass terms are a combination of right- and left-handed chiralities, since the right-handed piece of the neutrino does not participate in any interaction, it is assumed not to exist. In the absence of the right-handed component, the neutrino cannot have a Dirac mass.

Besides leptons, one also has the hadrons. Hadrons are divided into two categories, the mesons (pions, rhos, etc.) and baryons (protons, neutrons, etc.). In 1964, Gell-Mann and Zweig [Gel.64, Zwe.64] proposed that the hadrons were made of constituent quarks. The mesons were made from a quark-anti-quark pair and the baryons from a three-quark combination. This was to account for the apparent groupings of these particles in SU(3) representations.

Physical evidence for quarks came from deep inelastic ep scattering at SLAC in the latter half of the 1960's. Researchers there [Bre.69] measured the differential cross section, $d^2\sigma/(d\Omega dE')$, where E' is the energy of the outgoing electron. Dividing this by the Mott cross section to remove the major part of the well known momentum-transfer (q^2) dependence, they found that the resulting ratio had an unexpectedly weak q^2 dependence for excitations well beyond the resonance region. This weak q^2 dependence

was indicative of scattering off point-like constituents of the protons.

To rescue the three-quark baryons from apparently breaking the Pauli exclusion principle, three colours of quarks were suggested to exist. Experimental evidence soon backed this up. This was integral in the development of QCD as the theory for the strong interaction, of which I will talk shortly. There is evidence that six types of quarks exist, corresponding to three different flavours. Three charge $-1/3$ quarks, the down, strange and bottom quarks, and three charge $2/3$ quarks, the up, charm and top quarks. As of yet, the top quark has not been found. Free quarks apparently do not exist in nature, but must come in baryonic or mesonic combinations. Unlike leptons, flavour-changing interactions of the quarks do exist. Since this is a rather important component of this thesis, a discussion of this will come later.

One final aspect of this discussion on the matter content of the Standard Model is that recent findings from the CERN high luminosity e^+e^- collider LEP have found that the width of the Z^0 particle only allows three flavours of zero-mass neutrinos. Unless further flavours of neutrinos are massive, there would appear to be only three generations of matter. The full matter content of the Standard Model is given in figure 1.1.

Leptons			Quarks				
electron	0.511 MeV	electron neutrino	$< 17 \text{ eV}$	down	$\approx 7 \text{ MeV}$	up	$\approx 4 \text{ MeV}$
muon	105.7 MeV	muon neutrino	$< 0.27 \text{ MeV}$	strange	$\approx 150 \text{ MeV}$	charm	$\approx 1.6 \text{ GeV}$
tau	1.784 GeV	tau neutrino	$< 35 \text{ MeV}$	bottom	$\approx 4.9 \text{ GeV}$	top	$> 89 \text{ GeV}$

Figure 1.1: The matter content of the Standard Model. The masses of the low mass quarks are approximate as they are not free quarks.

As our final piece of the Standard Model, one has the strong interactions. The three colours of quarks serve as a basis of a colour symmetry group, $SU(3)_c$. This group is distinct from the $SU(3)$ flavour group that I mentioned earlier as being evidence for quarks. The strong force is then mediated by an $SU(3)$ octet of vector gluons. The gluons

are massless. Not only do these gluons mediate the strong force between quarks, but they also mediate it between the gluons themselves, since gluons have a colour charge, a product of the non-Abelian nature of the theory. This theory, known as QCD, has the unique property of asymptotic freedom. That is, the effective strong coupling depends on the momentum transfer, q^2 . This coupling is small for large q^2 and becomes large for small q^2 . This property prevents the escape of low mass, $q^2 \approx 0$, quarks and gluons and explains the lack of free quarks and gluons in nature.

The Standard Model is now complete, it is composed by attaching QCD to electroweak theory, for an $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ theory that is broken to a $SU(3)_c \otimes U(1)_{em}$ by the Higgs mechanism. This Higgs mechanism also provides a method for acquiring masses for the quarks and leptons through scalar, or Yukawa couplings, of the Higgs to the quarks and leptons. A full listing of the vector and scalar components is given in figure 1.2. There is no relationship between the strong coupling constants and the electroweak coupling constants. This is a matter for unification, which will be discussed later. Before we do that, two topics of importance should be discussed, chirality and flavour-changing interactions.

Vector Bosons	photon, γ	massless	electromagnetism
	gluons	massless	QCD
	$W^\pm$ Z^0	80.6 GeV 91.16 GeV	} weak interaction
Scalar	Higgs, H^0	> 49 GeV	symmetry breaking

Figure 1.2: The vector boson and scalar content of the Standard Model.

1.3 Chirality and Charge Conjugation

Dirac spinors can be split into left and right chiral pieces, $\psi_L = L\psi$ and $\psi_R = R\psi$, where $L = \frac{1}{2}(1-\gamma_5)$ and $R = \frac{1}{2}(1+\gamma_5)$. Under these projections, gauge interactions conserve

chirality, since (see Appendix A): $\bar{\psi}\gamma_{\mu}\psi = \bar{\psi}_L\gamma_{\mu}\psi_L + \bar{\psi}_R\gamma_{\mu}\psi_R$, whereas mass or Yukawa terms flip chirality: $\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$. This means that a particular gauge interaction could be completely left- or right-handed. Such is the case for the weak interaction. The exchange of a W involves only the left-handed portion. As a result it is customary to write the left-handed portion of leptons and quarks in terms of left-handed isodoublets; $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ and $\begin{pmatrix} u \\ d \end{pmatrix}_L$. Since Dirac mass terms involve a chiral flip, massive fermions require both a left- and right-handed piece. Thus in the Standard Model, one has e_R , u_R and d_R . Since neutrinos do not have mass in the Standard Model, no ν_R term is necessary.

The operation of charge conjugation will play an important part in what follows. In words, charge conjugation flips a particle into its anti-particle, or mathematically, $\psi^c \equiv C(\bar{\psi})^T$. Acting on chiral spinors, one has: $\psi_{L,R}^c \equiv C(\bar{\psi}_{R,L})^T$. This last statement implies that any right-handed state can be represented by the charge conjugate of a left-handed state. Thus e_R (or more precisely, $\bar{e}_R$) can be replaced by e_L^c , and similarly for the right-handed quark states. In supersymmetric theories it is customary to do precisely this, that is replace all right-handed states with left-handed conjugate states.

1.4 Flavour-Changing Interactions

It is experimentally well known that quarks have flavour-changing interactions, while leptons conserve flavour to a high degree of accuracy. Flavour-changing interactions occur in the Standard Model because the gauge and mass eigenstates are not simultaneously diagonalizable. To see this it is convenient to go to the gauge eigenstate of quarks, where the gauge interaction is considered to be flavour symmetric. The charged weak current is then:

$$J_{q\mu}^+ = \bar{q}_L^i \gamma_{\mu} \tau^+ q_L^i = \bar{p}_L^i \gamma_{\mu} n_L^i, \quad (1.1)$$

where i is a generation, or flavour index,

$$q_L^i = \begin{pmatrix} p_L^i \\ n_L^i \end{pmatrix} = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}, \begin{pmatrix} c_L^i \\ s_L^i \end{pmatrix}, \begin{pmatrix} t_L^i \\ b_L^i \end{pmatrix}, \quad (1.2)$$

is an isospin doublet, and $\tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is the $I=1$ isospin raising operator. One can now write down the mass term for the quarks:

$$\mathcal{L}_M = \frac{v}{\sqrt{2}} [(g_{Yuk}^D)_{ij} \bar{n}_L^i n_R^j + (g_{Yuk}^U)_{ij} \bar{p}_L^i p_R^j], \quad (1.3)$$

where v is the Higgs vacuum expectation value (vev), and g_{Yuk} are the Yukawa couplings for the down and up type quarks. The mass of the quarks is then given by:

$$M_{ij} = \frac{v}{\sqrt{2}} (g_{Yuk})_{ij}. \quad (1.4)$$

There is no a priori reason why this mass matrix must be diagonal. Indeed it should be a general 3 by 3 complex matrix in flavour space. Such a matrix can be diagonalized by two unitary transformations, i.e. $S^\dagger M T = M_d$, where M_d has positive eigenvalues. Since the mass eigenstate would be the states in which the mass matrix is diagonal, one has:

$$\bar{\psi}_L M \psi_R = (\bar{\psi}_L S) (S^\dagger M T) (T^\dagger \psi_R) = \bar{\psi}_L M_d \psi_R, \quad (1.5)$$

where $\psi_L = S^\dagger \psi_L'$ and $\psi_R = T^\dagger \psi_R'$ are the mass eigenstates. Returning to the charged weak current, one has in the mass eigenstate that

$$J_{q\mu}^+ = \bar{p}_L^i \gamma_\mu n_L^i = \bar{p}_L^i \gamma_\mu [S_{(p)}^\dagger S_{(n)}]_{ij} n_L^j = \bar{p}_L^i \gamma_\mu K_{(pn)}^{ij} n_L^j, \quad (1.6)$$

where K is a unitary matrix known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix [Kob.73]. In three flavour theory, this matrix has three real rotational angles and one complex phase factor. Thus the gauge interactions are not flavour diagonal, but rather via the charged weak current, quarks of a particular generation and isospin can be changed into quarks of another generation with opposite isospin. Notice that the equivalent of the CKM matrix for a neutral current would be $U = S_{(p)}^\dagger S_{(p)}$, which is obviously the unit matrix. There can be no flavour-changing neutral currents (FCNC), at least at the tree

level. Rather, all FCNC must proceed through loop diagrams.

Now how about leptons? For leptons, neutrinos do not have mass, therefore $S_{(v)}$ is arbitrary. In particular, one can pick $K_{(vl)} = S_{(v)}^\dagger S_{(l)} = \mathbf{1}$. In essence, since there is no mass eigenstate for the neutrino, what you define as a particular lepton anti-neutrino, is just that neutrino which is produced in association with the corresponding lepton. In the Standard Model, there are no flavour-changing interactions among the leptons.

This previous argument does not depend on the masslessness of the neutrino, but rather that all neutrino flavours are degenerate. If one could consider all quark masses of one type, be it up or down type, to be equal, then flavour-changing interactions between quarks would also disappear. This is known as the GIM mechanism. Mathematically it is $\sum_i K_{ip}^* K_{ip'} = 0$ for $p \neq p'$ and 1 for $p = p'$, which is just a property of the matrix unitarity. This is important, as it means any flavour-changing amplitude must have some quark mass dependence. In particular, if we look at the $K_L - K_S$ mass difference, one has that

$$\frac{m_L - m_S}{m_K} \approx \frac{2}{3} \frac{G_F}{\sqrt{2}} f_K^2 \frac{\alpha}{4\pi} \left(\frac{m_c^2 - m_u^2}{M_W^2 \sin^2 \theta_W} \right) \sin^2 \theta_c \cos^2 \theta_c, \quad (1.7)$$

where θ_c , the Cabibbo angle, is the one rotational angle in the two flavour equivalent of the CKM matrix. If the GIM cancellation did not take place, the $(m_c^2 - m_u^2) / M_W^2$ portion would be replaced by a term of order one. Realizing this fact allowed Glashow, Iliopoulos, and Maiani (the GIM in GIM) to propose the charmed quark [Gla.70] long before it was found experimentally. This is only the beginning of what flavour-changing interactions can do for us!

1.5 Grand Unification

As we have just seen, the Standard Model does a fine job explaining flavour-changing interactions through the CKM matrix. This matrix also allows an explanation of CP-violation through the one allowed phase. Explaining experimental results seems to be

a forté of the Standard Model, at least to this point. So why is it believed the Standard Model is incomplete. For one, it lacks an explanation of the various parameters. One has 3 coupling constants, 3 lepton masses, 6 quark masses, one Higgs vev, one Higgs mass, 3 CKM matrix angles and one CKM matrix phase for a total of 18 parameters. (The W and Z masses are determined by the SU(2) and U(1) coupling constants and the Higgs vev.) All these parameters must be experimentally determined. Wouldn't it be nice if all the parameters were determined, or at least some of them? I have already mentioned that the strong coupling constant depends on the energy scale, increasing in strength as $q^2 \rightarrow 0$. This is a general property of all non-Abelian theories; for small numbers of generations of matter, the coupling strength increases in strength as $q^2 \rightarrow 0$. For both an SU(3) and an SU(2) theory with three generations of matter, the coupling strength increases as $q^2 \rightarrow 0$. The opposite is true for Abelian, or U(1) theories, where the coupling constant decreases as $q^2 \rightarrow 0$. It therefore might be possible for the couplings (properly normalized) to be equal at some large energy scale. It is then just one simple step to say that the three couplings are different manifestations of a single coupling constant. For an $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ theory, in terms of some common coupling at a large energy scale, M_X , which I will call the GUT scale for reasons to become obvious, one has [Cam.90, Ros.85]:

$$\frac{1}{\alpha_c} = \frac{1}{\alpha_{GUT}} + \frac{1}{6\pi} (4N_G - 33) \ln\left(\frac{M_X}{E}\right), \quad (1.8a)$$

$$\frac{\sin^2\theta_W}{\alpha_{em}} = \frac{1}{\alpha_L} = \frac{1}{\alpha_{GUT}} + \frac{1}{6\pi} (4N_G - 22 + \frac{1}{2}) \ln\left(\frac{M_X}{E}\right), \quad (1.8b)$$

$$\frac{3}{5} \frac{\cos^2\theta_W}{\alpha_{em}} = \frac{1}{\alpha_Y} = \frac{1}{\alpha_{GUT}} + \frac{1}{6\pi} (4N_G + \frac{3}{10}) \ln\left(\frac{M_X}{E}\right), \quad (1.8c)$$

to one loop order in the renormalization group equations, with N_G being the number of fermion generations ($N_G=3$). Using the known values of the strong and electromagnetic couplings at the W mass scale, one finds that they meet at a scale $M_X \approx 10^{15}$ GeV.

Fiddling with two loop contributions to the renormalization group equations, one gets an M_X between 10^{14} and 10^{15} GeV. The above system is over constrained, we have only two unknowns, α_{GUT} and M_X , thus one can also use equation (1.8) to find a "theoretical" value for $\sin^2\theta_W$. Doing this we find $\sin^2\theta_W$ to be between 0.20 and 0.22, just below its experimental value. This is Grand Unification in its simplest form.

Grand Unification is just the idea that the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge interactions are unified into one simple gauge group, G , at the scale at which the gauge couplings are all equal, an idea first proposed by Pati and Salam [Pat.73]. The simplest such group that the Standard Model gauge interactions fit into is $SU(5)$ [Geo.74]. It has the added benefit that the Standard Model matter content can be placed nicely into $\underline{5}$ and $\underline{10}$ representations of $SU(5)$. The vector boson content is contained in the adjoint representation of $SU(5)$, which allows for 24 vector bosons. Since the Standard Model contains 12 vector bosons (8 gluons, 1 photon, 2 W's and 1 Z), we are left with 12 new bosons. These new bosons are given order M_X masses when $SU(5)$ is broken to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$.

Besides determining the gauge couplings, grand unified theories also predict ratios of quark and lepton masses. Since down quarks and leptons acquire mass through the same Higgs vev, their masses at the grand unified scale should be equal. One can then use the group renormalization equations to see what their masses should be at low energies. Using the mass of the tau particle, one can predict a mass for the b quark of order 5 GeV, quite close to the actual value. Unfortunately, this does not work as well for the other generations predicting a moderately too high (order 2-3) strange mass, and a far too small down quark mass. This situation can be somewhat corrected by complicating the Higgs sector.

It is the new bosons, typically referred to as X and Y bosons, that can lead to the

most interesting physics despite their huge mass, as they mediate baryon number violating processes. They allow proton decay! Typically the proton's lifetime depends on the fourth power of the X and Y masses, and when the dust settles for the simplest SU(5) models, one can obtain a proton lifetime of:

$$\tau_p = 0.2 \times 10^{29 \pm 2} \text{ yrs} \quad (1.9)$$

In particular, the partial width to the favoured mode, $p \rightarrow e + \pi^0$, is given by $\Gamma^{-1}(p \rightarrow e + \pi^0) = 4.5 \times 10^{29 \pm 1.7}$. The decay of the proton to this particular mode, as measured by the IMB collaboration [Par.85], now exceeds 10^{32} years, in clear contradiction with the SU(5) predictions.

The SU(5) estimates for proton decay can be changed somewhat by the complicated Higgs sector needed to explain fermion masses, however, it would still be in trouble from the experimental results. One can also go to more complicated groups such as SO(10), E(6), E(8), etc. Among the interesting possibilities in these theories are neutrino masses, extra low mass gauge bosons (a Z' or $W_R^\pm$), and an explanation of the number of generations. However, grand unified theories have some theoretical problems, the most important of which is the gauge hierarchy problem.

1.6 The Gauge Hierarchy Problem

As you may have noticed from the last section, a typical grand unified theory consists of a low mass sector with masses 200 GeV and below, plus a high mass sector around 10^{15} GeV. There is apparently nothing in between, and one has a jump in energy of 13 orders of magnitude between physics scales. There are many who believe this is not natural. Even if one is comfortable with this jump in energy, one must also look at possible renormalization corrections to the theory. As you have already seen for the gauge couplings, they receive corrections due to a high mass scale of order:

$$\delta\left(\frac{1}{\alpha(E)}\right) = O\left(\frac{1}{6\pi}\right) \ln\left(\frac{\Lambda}{E}\right), \quad (1.10)$$

where Λ is the scale of new physics. Natural log corrections are considered natural. Even at its worst, one would not expect corrections to the coupling constants of order

$$\delta\left(\frac{1}{\alpha(E)}\right) \gg O\left(\frac{1}{\alpha(E)}\right). \quad (1.11)$$

Fermion masses are much the same, they receive corrections due to a high mass scale of order:

$$\delta(m_f) = O\left(\frac{\alpha}{\pi}\right) m_f \ln\left(\frac{\Lambda}{E}\right). \quad (1.12)$$

Once again, corrections of order $\delta(m_f) \gg m_f$ would not occur, even if the scale of new physics was the Planck scale at 10^{19} GeV.

A scalar particle like the Higgs is different. Let's consider the various possible corrections to the Higgs mass at the one loop level, all of which are quadratically divergent. The contribution from a gauge boson would be



$$\delta(m_H^2) = g^2 \int_0^\Lambda \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} = O\left(\frac{\alpha}{\pi}\right) \Lambda^2, \quad (1.13)$$

where Λ is the cutoff. A fermion loop, with Yukawa coupling g_Y , would give similar results, except for a minus sign due to the fact we are dealing with a fermion loop:



$$\delta(m_H^2) = -g_Y^2 \int_0^\Lambda \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} = -O\left(\frac{\alpha_Y}{\pi}\right) \Lambda^2. \quad (1.14)$$

Finally, the Higgs mass could receive a correction from a scalar particle with coupling, g_S :



$$\delta(m_H^2) = g_S^2 \int_0^\Lambda \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} = O\left(\frac{\alpha_S}{\pi}\right) \Lambda^2. \quad (1.15)$$

Obviously, a scalar particle can have huge corrections to its mass. One can argue that the theory is still perfectly renormalizable in the normal way, but if the Higgs is to be embedded in a grand unified theory, the situation is even worse. For instance, in the SU(5) theory, SU(5) breaks down to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ via a large scale Higgs, ϕ , somewhere around 10^{15} GeV, and then breaks again to $SU(3)_c \otimes U(1)_{em}$ via small scale Higgs, H, at the weak scale, 10^2 GeV. Renormalization of the theory requires the existence of the coupling $\lambda H^2 \phi^2$ as a counterterm since terms of this kind will be introduced through divergent loop diagrams such as those of figure 1.3. At tree level a $\lambda H^2 \phi^2$ term would introduce a mass shift of

$$\delta m_H^2 = \lambda \langle \phi \rangle^2 = O(M_X^2) = O(10^{15} \text{ GeV})^2. \quad (1.16)$$

One could expect to regain a light Higgs mass by adding a negative bare mass of $-O(M_X^2)$, but diagrams like figure 1.3 would again shift the mass by:

$$\delta m_H^2 = O\left(\frac{\alpha}{\pi}\right) M_X^2 \approx O(10^{14} \text{ GeV})^2, \quad (1.17)$$

re-introducing the problem.

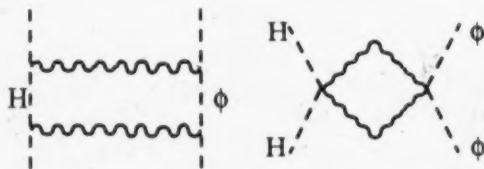


Figure 1.3: Feynman diagrams leading to huge electroweak Higgs, H, mass corrections in SU(5)

Even if grand unified theories did not exist, one might still have a problem since Hawking, Page and Pope [Haw.80] argue that elementary scalars propagating through the space-time foam which bubbles up at the Planck scale due to quantum gravity effects have their masses shifted by:

$$\delta m_H^2 = \lambda O(M_P^2) = O(10^{19} \text{ GeV})^2. \quad (1.18)$$

Unless one believes in these very delicate and unnatural cancellations, one must find a way out of this dilemma. Since scalar particles seem to be a problem, one route out is to eliminate the scalars taking a hint from the BCS theory of superconductivity, where the scalar field used for symmetry breaking is actually a bound state of two fermions. Any corrections to this would be cutoff at the binding scale, which should be around 1 TeV. This possibility is the strategy employed by theories like Technicolour. The second solution is to notice the difference in signs between equations (1.13) and (1.15) and equation (1.14). If one had a scalar particle of equal mass for every fermion, and a fermion of equal mass for every gauge particle, one would have an exact cancellation of divergent terms. This is known as supersymmetry.

1.7 Technicolour

The actual theory of technicolour will not be important to this thesis. It is however a very elegant theory that will lead us into a discussion of composite models. Furthermore, the subject of flavour-changing interactions plays an important role in constraints on technicolour theories. As such, a short discussion is in order.

As I mentioned earlier, technicolour replaces the scalar Higgs with a bound state of two fermions, motivated by the Cooper pair of electrons in BCS superconductivity. It is further motivated by the fact that the W and Z would have obtained masses due to the chiral symmetry breaking of the strong interaction. Of course this mass is phenomenologically far too small, $M_W \approx 30 \text{ MeV}$, but if one had a strong like interaction at a higher scale, one could produce a phenomenologically acceptable model. Let's look at this mechanism.

Consider a doublet of quarks containing an up and a down quark, interacting through QCD. The Lagrangian of the theory has an $SU(2)_L \otimes SU(2)_R \otimes U(1)$ symmetry.

The Lagrangian is invariant under the transformations $Q'_{L,R} = U_{L,R} Q_{L,R} = U_{L,R} \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$. This symmetry is broken by a fermion condensate,

$$\langle \bar{u}u + \bar{d}d \rangle = \langle \bar{Q}Q \rangle = \langle \bar{Q}_L Q_R + \bar{Q}_R Q_L \rangle. \quad (1.19)$$

The vacuum is not invariant under independent transformations U_L and U_R , however, it is invariant under the transformation $U_L = U_R$. In other words, the vacuum is invariant under $SU(2)_{\text{isospin}}$, as is expected.

Goldstone's theorem now says that 3 massless spin-0 Goldstone bosons should be produced when this symmetry is broken, these 3 bosons are the pions, π_i , which form an $SU(2)$ isotriplet. The pion decay constant, given by $\langle 0 | J_{5i}^\mu | \pi_j \rangle = f_\pi q^\mu \delta_{ij}$, together with the electroweak interactions,

$$\mathcal{L}_{EW} = \frac{1}{2} g_2^2 W_{\mu\pm 5}^\pm J_{\mu\pm 5}^\pm + g_2 W_{\mu 05}^0 J_{\mu 05}^0 + g_1 B_{\mu 05}^0 J_{\mu 05}^0, \quad (1.20)$$

give $M_W = \frac{1}{2} g_2 f_\pi$, $M_\gamma = 0$, and $M_Z = \frac{1}{2} (g_1^2 + g_2^2)^{1/2} f_\pi$. The ratio of the W and Z masses are just what one expects:

$$\frac{M_W}{M_Z} = \frac{g_2}{(g_1^2 + g_2^2)^{1/2}} = \cos \theta_W. \quad (1.21)$$

There is one problem, the experimental value of f_π gives a W mass of 30 MeV. Furthermore, the pions exist as physical particles, and not just one more degree of freedom of the W and Z. Perhaps there exists another QCD-like interaction, an $SU(N)$ ($N > 3$) technicolour interaction with techniquarks U and D that have an $SU(2)_L \otimes SU(2)_R \otimes U(1)_{L+R}$ symmetry broken by condensates,

$$\langle \bar{U}_L U_R \rangle = \langle \bar{U}_R U_L \rangle = \langle \bar{D}_L D_R \rangle = \langle \bar{D}_R D_L \rangle \approx O(\Lambda_{TC}^3) \approx O(1 \text{ TeV})^3, \quad (1.22)$$

giving three technipion goldstone bosons. To reproduce the desired W and Z masses, one would require a technipion decay constant, F_π , of 250 GeV.

Although the technipions would have disappeared, becoming the longitudinal degrees of freedom of the W and Z, there would be other techni-hadrons produced, the equivalents of the η , ρ and ω ; as well as techni-baryons. Actually, to be able to produce

quark and lepton masses, one requires an $SU(8)_L \otimes SU(8)_R$ chiral symmetry broken to $SU(8)_{L+R}$, producing 63 goldstone bosons plus one singlet state, of which only three disappear to produce W and Z masses. At least some of these other states should have low masses and be experimentally observable.

The more interesting problem of technicolour theories, at least as far as this thesis is concerned, is how quark and lepton masses are produced. To produce fermion masses, one requires a four-fermion coupling:

$$\mathcal{L}_{Yuk}^{TC} = g(\bar{f}f)(\bar{F}F), \quad (1.23)$$

where F is a technifermion, and g is the coupling constant, which must have dimension one over mass squared. This coupling is unrenormalizable, and thus one would expect it to be an effective coupling for the exchange of either a scalar or a gauge boson. Since the theory was invented to get rid of the problems associated with scalars, the gauge boson is the logical choice. Producing fermion masses through the exchange of a vector boson, as in figure 1.4, is known as extended technicolour. Since the extended technicolour group commutes with $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, one requires a whole family of technifermions (i.e. 2×3 techniquarks plus 2 leptons - the 8 in $SU(8)_L \otimes SU(8)_R$ mentioned earlier). With a vector boson exchange one obtains a fermion mass:

$$m_f = \frac{g_{Vf}^2}{m_{Vf}^2} \langle 0 | \bar{F}F | 0 \rangle = O(\Lambda_{TC}^3) \frac{g_{Vf}^2}{m_{Vf}^2}. \quad (1.24)$$

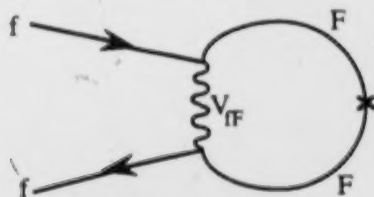


Figure 1.4: Fermion mass generating diagram through vector boson exchange.

To obtain masses for several generations, one requires a whole set of gauge bosons, $V_{f_i F_j}$, where $j=1\dots 3$ is a generation index, and $i=1\dots N$ is a technicolour index. Since successive couplings of $V_{f_i F_j} V_{F_j f_i}$ will couple f_j to f_j , one also requires the gauge bosons, $V_{f_i f_i}$, to close the algebra. In order to achieve the observed fermion mass spectrum, extended technicolour must break in stages, giving different masses to the different gauge bosons. To generate an acceptable pattern of fermion masses, one requires masses of $m_{V_{A_P}}^2 \approx 10^3 (\text{TeV})^2$ for the first generation, $m_{V_{B_P}}^2 \approx 30 (\text{TeV})^2$ for the second generation, and $m_{V_{C_P}}^2 \approx 1 (\text{TeV})^2$ for the third generation.

The real problem lies in these $V_{f_i f_i}$ gauge bosons needed to close the algebra. They will introduce flavour-changing neutral currents with an effective lagrangian,

$$\begin{aligned} \mathcal{L}_{(sd)}^{\text{eff}} &= \frac{g_{V_{A_P}}^2}{m_{V_{A_P}}^2} \cos^2 \theta \sin^2 \theta (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L) \\ &\approx \frac{1}{(m_{V_{A_P}}^2 \text{ or } m_{V_{B_P}}^2)} \cos^2 \theta \sin^2 \theta (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L) \end{aligned} \quad (1.25)$$

The values for $m_{V_{A_P}}^2$ and $m_{V_{B_P}}^2$ needed to produce the fermion masses produce a far too large K_L-K_S mass difference. Once again, flavour-changing interactions have told us much about the validity of a theory. Problems with flavour-changing interactions might be solved by "walking" or "stagnant" technicolour theories, but a discussion of this is beyond the scope of this thesis.

1.8 Compositeness of Quarks and Leptons¹

Once one has suggested a composite Higgs scalar, it is only a short step to consider composite fermions as well. The history of particle physics is full of instances where what were assumed to be elementary particles turned out to be composite

¹For a review of composite models of quarks and leptons, and of composite gauge interactions, see refs. [Pes.81, Pec.87]

structures. The latest example of this composite nature of thought to be elementary particles being the quark structure of hadrons. The possibility of substructure for quarks and leptons is seemingly quite attractive, since it possibly might provide a solution to the generation problem, just as quarks provided a solution to the multitude of hadronic states that were being discovered at that time. There would seem to be one problem with this concept, however. As we will see below, experimental evidence suggests that the scale of quark and lepton compositeness, if it exists, must lie above 1 TeV, yet the masses of the quarks and leptons are far below this scale. This should be compared with other examples of compositeness: In the case of hadronic matter, the mass of the hadrons is of the same order as the composite scale, Λ_{QCD} . For a loosely bound structure like the positronium (an e^+e^- bound state or "atom"), the mass of the system far exceeds the binding scale. Obviously, composite quarks and leptons must be quite different from these two cases. Possible solutions to this would undoubtedly involve invoking some symmetry, unbroken at the composite scale, that forbids mass terms. Since fermion mass terms involve left and right chiral pieces, the logical candidate for such a symmetry would be a chiral symmetry. Any gauge interaction respects a chiral symmetry, however, it must do more than that. I have already discussed how QCD respects a chiral symmetry, but still allows for the formation of quark-anti-quark condensates that spontaneously break this symmetry. A composite theory of quarks and leptons with constituent preons must somehow prevent mass generation for composite fermions from vacuum condensates.

The spectrum of massless composite fermions that can be produced is severely constrained by an anomaly matching condition first noticed by 't Hooft. 't Hooft's observation ['tHo.80] was that the underlying theory with chiral symmetries might be subject to anomalies, and in particular, the Adler-Bell-Jackiw anomaly [Adl.69, Bel.69].

These anomalies may be seen as giving contributions to the correlation functions at zero frequency. As such these properties of the correlation functions appear in the long distance, or infrared, behavior, and must be reproduced by the effective low energy theory. In other words, the calculation of the anomaly using either the elementary preons or the massless composite states must give the same answer, as shown in figure 1.5. Given an underlying preon theory, this imposes severe constraints on the composite fermion spectrum it might produce.

$$\sum_{\text{elementary fermions}} \text{[triangle diagram]} = \sum_{\text{massless composite fermions}} \text{[triangle diagram]}$$

Figure 1.5: Restriction on composite fermions due to t' Hooft anomaly condition.

There is another possibility to protect the composite quarks and leptons from acquiring a large mass. It is known as the quasi-Nambu-Goldstone fermion mechanism. If the preon theory has some symmetry that is broken down to a smaller symmetry at the binding scale, then Goldstone bosons emerge in the bound state spectrum. If the theory is supersymmetric, these Goldstone bosons will be accompanied by fermionic partner states, (quasi-Nambu-Goldstone fermions) which might be identified with the quarks and leptons. Since this concept involves supersymmetry, and is also the motivation for much of the work done in this thesis, a further discussion of it will be pursued in Chapter 2.

If quarks are indeed composite, how would this exhibit itself experimentally. One could always do scattering experiments to see if the interactions between the constituent particles start to manifest themselves by introducing interference terms between the new composite scale interactions and the longer range interactions. This strategy worked well for finding the constituent quarks in protons, or indeed even in Rutherford's classic demonstration of the existence of the short range nuclear force by scattering alpha

particles off gold nuclei. Parametrizing the contact force as a dimension-six operator,

$$\mathcal{L}_{\text{contact}} \approx \frac{1}{M_c^2} [\bar{f}f\bar{f}f], \quad (1.26)$$

where M_c is the scale at which compositeness manifests itself, one can set limits on the scale of order $M_c \geq 1$ TeV from scattering experiments at PETRA..

If one was able to do experiments at the composite scale, one would undoubtedly find new particles such as excited quarks and leptons, the equivalents of such states as the ρ^* and η^* in meson spectroscopy. The search for such states would undoubtedly be done at future colliders such as the SSC.

Such experimental tests are restricted by the fact that to see new particles one needs to have a collider operating at the composite scale. Even to see the interference between the composite scale forces and longer range forces, one needs to get up to energies where the new composite interactions are significant. This can be avoided if one looks at interactions that do not respect the conservation laws that are built into the Standard Model and therefore do not compete against Standard Model contributions. In general one might expect that the interchange of the constituents in composite models may lead to effective interactions that violate baryon number, lepton number, or quark and lepton flavour (flavour-changing interactions). Since baryon number is conserved to a very high degree, one should expect that baryon and lepton number conservation should be built into the theory, however, one should expect flavour-changing interactions to occur. This would be especially true if one wanted to produce a theory where, say, the electron, muon and tau particle are built out of the same particles, the muon and tau particle being some sort of excited state of the electron. Here one would again have a four-fermion interaction, as in equation (1.26), except now the fermions need not be of the same generation. In a theory that respects a chiral symmetry, one has a limit from (g-2) of the muon of $M_c \geq 1$ TeV. For a non flavour-conserving theory, one has that

$$\Gamma(\mu \rightarrow e\gamma) = \frac{m_\mu^5}{16\pi M_C^4}; \quad \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow \text{all})} = \frac{12\pi^2}{G_F^2 M_C^4}, \quad (1.27)$$

which for a branching ratio for $\mu \rightarrow e\gamma$ of 4.9×10^{-11} gives a constraint of $M_C \geq 350$ TeV.

In what follows, we will see what type of constraints on compositeness follow from the rare processes of π_{21} decay, muon conversion, the K_L - K_S mass difference and from the neutron electric dipole moment. As well, we will show how to get even better constraints if one has a supersymmetric composite model.

1.9 Composite Gauge Bosons

Once one has gone so far as to discuss composite fermions, why not take this one step further and consider composite gauge bosons. A theorem discovered by Case and Gasiorowicz [Cas.62] and rediscovered and expanded by Weinberg and Witten [Wei.80] forbids the existence of massless composite particles with spins greater than 1/2. It does not forbid the existence of massive composite bosons, at least as long as their masses are of the order of the composite mass scale, M_C :

$$M_V \geq g M_C, \quad (1.28)$$

where g is a gauge coupling constant, implying $M_C \sim O(1 \text{ TeV})$ if one wishes to consider composite W and Z bosons. The advantage of having composite W 's and Z 's is that it eliminates any need for a Higgs mechanism and the problems associated with that.

It has long been known [Hun.78, Bjo.79] that the $q^2 \rightarrow 0$ limit of electroweak interactions in the Standard Model can be reproduced by a theory with a global $SU(2)_L$ invariance in which W^3 - γ mixing occurs. Starting with the Lagrangian:

$$\mathcal{L} = -\frac{1}{4}\tilde{F}_{\mu\nu} \cdot \tilde{F}^{\mu\nu} - \frac{1}{4}W_{\mu\nu} \cdot W^{\mu\nu} + \frac{1}{2}M_W^2 W_\mu \cdot W^\mu + e\tilde{A}_\mu j_{em}^\mu + gW_\mu \cdot j^\mu, \quad (1.29)$$

where

$$\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu, \quad W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu, \quad j_{em}^\mu = \sum_i \bar{\psi}_i \gamma_\mu Q_i \psi_i, \quad j_\mu = \sum_i \bar{L}_i \gamma_\mu \frac{1}{2} L_i. \quad (1.30)$$

L_i are the left-handed quark and lepton isodoublets with i running over all the quarks and leptons. The photon field has a tilde over it as the physical field will be a rotated version of the above photon field. To this lagrangian, one adds the following current mixing term:

$$\mathcal{L} = -\frac{1}{4} \lambda (\tilde{F}_{\mu\nu} \cdot W^{3\mu\nu} + W_{\mu\nu}^3 \cdot \tilde{F}^{\mu\nu}). \quad (1.31)$$

The resulting lagrangian can be diagonalized by the non-unitary transformation,

$$\begin{pmatrix} \tilde{A}_\mu \\ W_\mu \end{pmatrix} = \begin{pmatrix} 1 & \frac{-\lambda}{\sqrt{1-\lambda^2}} \\ 0 & \frac{1}{\sqrt{1-\lambda^2}} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}, \quad (1.32)$$

and produces the Standard Model neutral current lagrangian,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} \cdot Z^{\mu\nu} + \frac{1}{2} \frac{M_W^2}{1-\lambda^2} Z_\mu \cdot Z^\mu + e A_\mu j_{em}^\mu + \frac{g}{\sqrt{1-\lambda^2}} Z_\mu (j_3^\mu - \frac{\lambda e}{g} j_{em}^\mu), \quad (1.33)$$

provided $\lambda = \sin \theta_W = e/g$. The requirement $\lambda = e/g$ is known as the W dominance condition. One could now also add trilinear $SU(2)_L$ invariant vector boson interactions, that since we are dealing with a composite theory, need not be constrained to dimension-4 terms, but could also include dimension-six terms. Kuroda et al have investigated such a possibility [Kur.87]. We will review this in Chapter 5 and consider the implications of such couplings on $b \rightarrow s$ transitions.

1.10 Supersymmetry

One need not abandon elementary gauge bosons or even the Higgs mechanism to solve the gauge hierarchy problem. As mentioned earlier, the fermion corrections to the Higgs mass, equation (1.14), comes with the opposite sign of the gauge and scalar corrections, equations (1.13) and (1.15). Therefore, if for every fermion one had a scalar

particle of the same mass and same coupling constant to the Higgs; and for every gauge particle, one had a fermion of equal mass with the same coupling constant to the Higgs, there would be no correction to the Higgs mass. This is known as supersymmetry. Phenomenologically, this does not happen. There is no scalar electron with mass equal to that of the electron. Indeed, such a particle is known not to exist. One does not require such an exact cancellation, however, for corrections to the mass of a particle are considered natural if it is not too much greater than the uncorrected mass of the particle. Thus one has that

$$\delta m_H^2 = O\left(\frac{\alpha}{\pi}\right) |m_b^2 - m_t^2| \approx m_H^2. \quad (1.34)$$

Requiring a Higgs mass of order the weak scale means the difference between the scalar or gauge boson mass and the fermion mass should be no more than of $O(1 \text{ TeV})$. If supersymmetry is to be correct, we should find a whole new spectrum of particles around the TeV range, one for every known particle. Actually, one matches physical degrees of freedom: For every complex scalar, ϕ , (two degrees of freedom - particle and anti-particle) one requires a Weyl Fermion, ψ_L , (two degrees of freedom - a left-handed particle and a right-handed anti-particle). For each massless Hermitian gauge boson, A_μ^a , (two degrees of freedom - the two transverse polarizations) one requires a self-conjugate Majorana fermion, λ^a , (two degrees of freedom - spin parallel or anti-parallel). Lastly, one could also include a spin-2 graviton (two polarizations) which requires a spin-3/2 Rarita-Schwinger gravitino (two helicity states). The particle spectrum of the supersymmetric Standard Model is given in figure 1.6.

Before we go on, it should be explained what a Majorana fermion is. In sections 1.3 and 1.4 it was mentioned that fermions acquire a Dirac mass term through coupling of left and right chiralities, $\mathcal{L}_{\text{mass}} = m(\psi_L \psi_R + \psi_R \psi_L)$. A fermion with a Dirac mass term is known as a Weyl fermion. There is a way for a neutral charged fermion to acquire mass

even if it has only one chirality. It can do this by coupling to its own anti-particle through a coupling $(\psi_L)^T C \psi_L$, where C is the charge conjugation matrix. In essence, the particle has become its own anti-particle. Such a mass term is known as a Majorana mass term, and such a spinor, a Majorana spinor.

$J = 0$	$J = 1/2$	$J = 1$
squark, $\tilde{q} \geq 70 \text{ GeV}$ slepton, $\tilde{l} \geq 40 \text{ GeV}$ Higgs $\begin{cases} H^0 \\ H^{0'}, H^{0''} \\ H^\pm \geq 45 \text{ GeV} \end{cases}$	quark, q lepton, l Higgsino $\begin{cases} \tilde{H}^0 \dots \\ \tilde{H}^\pm \end{cases}$ gluino, $\tilde{g} \geq 70 \text{ GeV}$ photino, $\tilde{\gamma}$ Wino, $\tilde{W}^\pm \geq 45 \text{ GeV}$ Zino, $\tilde{Z}$	gluon photon $W^\pm$ Z

Figure 1.6: The particle spectrum of the Supersymmetric Standard Model, with the limits on the masses of the supersymmetric particles given.

1.11 The Algebra of Supersymmetry

So far we have talked about what supersymmetry can do for the gauge hierarchy problem, and how it would double the number of known particles. But what is supersymmetry? Supersymmetry is a symmetry that relates spin-0 and -1 bosons to spin-1/2 fermions. To do this, one requires a spin-1/2 Majorana fermion, Q , as the generator so that

$$Q|F\rangle = |B\rangle, \text{ and } Q|B\rangle = |F\rangle. \quad (1.35)$$

One really had no choice but to introduce a symmetry with a generator that transformed as a spinor, as a theorem by Coleman and Mandula [Col.67] states that the only conserved quantities that transform as tensors under the Lorentz transformation are the generators of

the Poincaré group, p_μ and $M_{\mu\nu}$. As such these SUSY operators along with the Poincaré group form a graded Lie algebra [Jog.84]:

$$[p_\mu, p_\nu] = 0, \quad (1.36a)$$

$$[M_{\mu\nu}, p_\rho] = i(g_{\rho\mu}p_\nu - g_{\rho\nu}p_\mu), \quad (1.36b)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -\delta_{\nu\mu}^{\alpha\beta}(M_{\alpha\rho}g_{\sigma\beta} - M_{\alpha\sigma}g_{\rho\beta}), \quad (1.36c)$$

$$[Q_\alpha, p_\nu] = 0, \quad (1.36d)$$

$$[Q_\alpha, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_{\alpha\beta}Q_\beta, \quad (1.36e)$$

$$\{Q_\alpha, Q_\beta\} = -(\gamma^\mu C)_{\alpha\beta}P_\mu. \quad (1.36f)$$

Equations (136a)-(136c) are just the standard Lie algebra associated with the generators, p_μ and $M_{\mu\nu}$, of the Poincaré group. Equation (136d) informs us that the Q 's, or supertranslations, are independent of translations in spacetime, whilst (136e) tells us that the Q 's transform as spinors under the Lorentz transformation. Finally, (136f) means that supersymmetric transformations induce translations on the states on which they act. Since Q is a Majorana fermion, it should be remembered that $Q_\alpha = C_{\alpha\beta}\bar{Q}_\beta$ (C being the charge conjugation matrix) and (137f) can be rewritten:

$$\{Q_\alpha, \bar{Q}_\beta\} = (\gamma^\mu)_{\alpha\beta}P_\mu. \quad (1.36f')$$

The transformation laws given above apply for $N=1$ supersymmetry (SUSY). $N=1$ SUSY is capable of changing spin by $1/2$. However, in gauge theories, changes of spin from -1 to 1 are allowed, opening up the possibilities of up to 4 SUSY charges, or $N=4$ SUSY. In supergravity theories, spins between -2 and 2 are allowed, and one can have $N \leq 8$ SUSY theories. In such a theory, (136f) is generalized to:

$$\{Q_\alpha^i, Q_\beta^j\} = -\delta^{ij}(\gamma^\mu C)_{\alpha\beta}P_\mu + C_{\alpha\beta}Z_{ij} + (\gamma_5 C)_{\alpha\beta}Z'_{ij}, \quad (1.37)$$

where the generators Z_{ij} and Z'_{ij} are antisymmetric in i and j . $N \geq 1$ theories are not phenomenologically acceptable as low energy effective theories, and therefore this is the last time they shall be mentioned in this thesis.

In what follows we will only be concerned with $N=1$ SUSY. To account for the particles we wish to have in the theory, we will require two supermultiplets, a gauge

supermultiplet, $V = (B_\mu, \chi)$, consisting of a spin-1 vector boson and a spin-1/2 Majorana fermion, and chiral supermultiplet, $\Phi = (\phi, \psi)$, consisting of a spin-0 scalar and a spin-1/2 Weyl fermion. The interactions of these two supermultiplets will be the subject of the next section.

1.12 The Supersymmetric Lagrangian

Let's consider the chiral supermultiplet, Φ^i ,

$$\Phi^i = (\phi^i, \psi_L^i, F^i), \quad (1.38)$$

where as previously mentioned; ϕ^i is a dimension 1 scalar field, ψ_L^i is a dimension 3/2 Weyl fermion and F , referred to as the auxiliary field, is a non-propagating dimension 2 scalar field. The auxiliary field will disappear after the equations of motion have been applied. In addition to the chiral supermultiplet, we have its complex conjugate,

$$\bar{\Phi}_i = (\phi_i^*, \bar{\psi}_{iR}^*, F_i^*). \quad (1.39)$$

Let us now investigate the properties of this chiral supermultiplet under the action of an infinitesimal SUSY transformation:

$$\Lambda(Q) = \epsilon \bar{\epsilon} Q, \quad (1.40)$$

where ϵ is an infinitesimal anticommuting Majorana spinor parameter. Using the relation (136f) it is easy to show this implies,

$$[\Lambda_1(Q), \Lambda_2(Q)] = [\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q] = \bar{\epsilon}_1 \gamma^\mu \epsilon_2 p_\mu, \quad (1.41)$$

and more specifically with $p_\mu = -i\partial_\mu$, acting on the supermultiplet one has:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \Phi^i = -i \bar{\epsilon}_1 \gamma^\mu \epsilon_2 \partial_\mu \Phi^i. \quad (1.42)$$

In order for the components of the chiral multiplet to obey this relation, it can be shown [DeW.75] that they must transform as:

$$\delta\phi^i = \bar{\epsilon}\psi_L^i \quad (1.43a)$$

$$\delta\psi_L^i = \frac{1}{2}(1 - \gamma_5)(F^i - i\partial\phi^i)\epsilon, \quad (1.43b)$$

$$\delta F^i = -i\bar{\epsilon}\partial\psi_L^i. \quad (1.43c)$$

In 1974, Wess and Zumino [Wes.74] constructed a renormalizable Lagrangian that obeyed (1.43). It is:

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{KE}} + \mathcal{L}_{\text{PE}} + \mathcal{L}_{\text{YUK}} \quad (1.44)$$

where

$$\mathcal{L}_{\text{KE}} = \bar{\psi}_L \not{\partial} \psi_L + \partial_\mu \phi_i^* \partial^\mu \phi^i + F_i^* F^i, \quad (1.44a)$$

$$\mathcal{L}_{\text{PE}} = \sum_i \left[\left(\frac{\partial W}{\partial \phi^i} \right) F^i + \left(\frac{\partial W}{\partial \phi^i} \right)^* F_i^* \right], \quad (1.44b)$$

$$\mathcal{L}_{\text{YUK}} = \frac{1}{2} \sum_{ij} \left[\frac{\partial^2 W}{\partial \phi^i \partial \phi^j} (\psi_L^i)^T C^{-1} \psi_L^j + \left(\frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \right)^* \bar{\psi}_L^j C (\bar{\psi}_L^i)^T \right], \quad (1.44c)$$

and the superpotential, W , is given by

$$W = c_i \phi^i + m_{ij} \phi^i \phi^j + g_{ijk} \phi^i \phi^j \phi^k. \quad (1.45)$$

The superpotential is restricted to being cubic in ϕ in order that the Lagrangian be renormalizable. This restriction will be lifted in Chapter 2.

Applying the equations of motion to the auxiliary field, one finds that

$$F_i^* + \frac{\partial W}{\partial \phi^i} = 0, \quad (1.46a)$$

and

$$F^i + \left(\frac{\partial W}{\partial \phi^i} \right)^* = 0. \quad (1.46b)$$

This allows us to eliminate F from the Lagrangian and we arrive at the final form of the Lagrangian:

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{KE}} + \mathcal{L}_{\text{PE}} + \mathcal{L}_{\text{YUK}} \quad (1.47)$$

where,

$$\mathcal{L}_{KE} = \bar{\psi}_L \partial \psi_L + \partial_\mu \phi_i^* \partial^\mu \phi^i, \quad (1.47a)$$

$$\mathcal{L}_{PE} = - \sum_i \left| \frac{\partial W}{\partial \Phi^i} \right|^2, \quad (1.47b)$$

$$\mathcal{L}_{YUK} = \frac{1}{2} \sum_{ij} \left[\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j} (\psi_L^j)^T C^{-1} \psi_L^i + \left(\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j} \right)^* \bar{\psi}_L^j C (\bar{\psi}_L^i)^T \right], \quad (1.47c)$$

To obtain a realistic theory, one would wish to make this a gauge theory. Consider an infinitesimal gauge transformation:

$$\Lambda(L) = e^{i\theta^a L_a}, \quad (1.48)$$

where the θ^a are the parameters of the Hermitian generators L_a^i , of the Lie group, G , and

$$[L_a, L_b] = i f_{abc} L_c, \quad (1.49)$$

where f_{abc} are the group structure constants. The chiral multiplet transforms as:

$$\delta \Phi = i \theta^a L_a \Phi \quad (1.50)$$

under this group. Needless to say, we should introduce our gauge supermultiplet,

$$V^a = (B_\mu^a, \chi^a, D^a), \quad (1.51)$$

where besides the dimension 1 vector boson, B_μ , and the dimension 3/2 Majorana spinor, χ , one also has the dimension 2 auxiliary scalar field, D . Each of these has its own transformation under G [DeW.75]. With these additions, the normalization of the supersymmetric transformations, equation (1.43), becomes:

$$[\delta_{\epsilon_1}, \delta_{\bar{\epsilon}_2}] = -i \bar{\epsilon}_1 \gamma^\mu \epsilon_2 D_\mu, \quad (1.52)$$

where

$$D_\mu \Phi = (\partial_\mu - i g L^a B_\mu^a) \Phi, \quad (1.53a)$$

$$D_\mu \bar{\Phi} = (\partial_\mu + i g L^{aT} B_\mu^a) \bar{\Phi}, \quad (1.53b)$$

$$(D_\mu \chi)^a = \partial_\mu \chi^a + g f_{abc} B_\mu^b \chi^c, \quad (1.53c)$$

$$(D_\mu B_\nu)^a = - F_{\mu\nu}^a, \quad (1.53d)$$

$$(D_\mu D)^a = \partial_\mu D^a + g f_{abc} B_\mu^b D^c \quad (1.53e)$$

and

$$F_{\mu\nu}^a = \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + gf_{abc} B_\mu^b B_\nu^c. \quad (1.54)$$

One can now work out what transformation laws the vector supermultiplet must have under SUSY transformations to satisfy equation (1.52) and then proceed to find a Lagrangian invariant under both SUSY and gauge transformations, at which point the auxiliary fields can be eliminated using the equations of motion. The Yukawa and potential energy Lagrangians of equation (1.44) are gauge invariant provided the superpotential has symmetric gauge covariant coefficients, c_i , m_{ij} , g_{ijk} . However, the kinetic energy Lagrangian is substantially changed. The final result, after eliminating the auxiliary fields is [DeW.75]:

$$\mathcal{L} = \mathcal{L}_{\text{GAUGE}} + \mathcal{L}_{\text{MATTER}} + \mathcal{L}_{\text{SCALAR}} + \mathcal{L}_{\text{YUKAWA}}, \quad (1.55)$$

where

$$\mathcal{L}_{\text{GAUGE}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{2} \bar{\chi}^a \gamma^\mu (D_\mu \chi)^a, \quad (1.55a)$$

$$\mathcal{L}_{\text{MATTER}} = (D_\mu \phi)^\dagger (D_\mu \phi) + \frac{1}{2} \bar{\psi}_L \gamma^\mu D_{\mu j}^i \psi_L^j + i\sqrt{2} g \bar{\chi}^a \phi_i T_j^{ai} \psi_L^j + i\sqrt{2} g \bar{\psi}_L T_j^{ai} \phi^j \chi^a, \quad (1.55b)$$

$$\mathcal{L}_{\text{YUKAWA}} = \frac{1}{2} \sum_{ij} \left[\frac{\partial^2 W}{\partial \phi^i \partial \phi^j} (\psi_L^j)^\dagger C^{-1} \psi_L^i + \left(\frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \right)^* \bar{\psi}_L C (\bar{\psi}_L^j)^\dagger \right], \quad (1.55c)$$

$$\mathcal{L}_{\text{SCALAR}} = -\sum_i \left(\frac{\partial W}{\partial \phi^i} \right)^* \left(\frac{\partial W}{\partial \phi^i} \right) - \frac{1}{2} \sum_a (g \phi_i T_j^{ai} \phi^j - \xi^a)^2, \quad (1.55d)$$

and ξ^a appears only when $f_{abc} \xi^a = 0$, for all b and c (i.e. one has an invariant Abelian subalgebra). Notice that in addition to the standard gauge interactions, one also has gauge-like interactions involving one chiral fermion, a chiral scalar and a gauge Majorana fermion. Some of the relevant Feynman rules resulting from this Lagrangian can be found in Appendix B.

1.13 Supersymmetric Masses and Flavour-Changing Interactions

The masses of the quarks and leptons come from the Yukawa term of the Lagrangian. The complete superpotential for the Supersymmetric Standard Model (SSM), which is just a supersymmetric version of $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, is:

$$W = g_{Yuk}^d D_L^c Q_L H_L + g_{Yuk}^e E_L^c L_L H_L + g_{Yuk}^u U_L^c Q_L H_L' \quad (1.56)$$

where Q_L and L_L are respectively the left-handed quark and lepton isodoublet superfields, D_L^c , U_L^c , E_L^c are the down, up and lepton conjugate-left superfields, and H_L and H_L' are the Higgs isodoublet superfields. The isodoublets are given by

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad L_L = \begin{pmatrix} N_L \\ E_L \end{pmatrix}, \quad H_L = \begin{pmatrix} H_L^0 \\ H_L^- \end{pmatrix}, \quad H_L' = \begin{pmatrix} H_L^{+'} \\ H_L'^0 \end{pmatrix} \quad (1.57)$$

with the contraction rule $Q_L L_L = U_L E_L - D_L N_L$, etc. The two Higgs doublets are necessary as the superpotential only links fields to other fields, and does not link fields to the complex conjugates of other fields, so one cannot produce all the quark and lepton masses with one scalar. Rather, one must have two Higgs isodoublets, H_L , which gives mass to the down quarks and leptons, and H_L' , which gives mass to the up quarks. A full list of superfield $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers is given in Appendix C.

We are interested in the terms of the superpotential like

$$W \supset g_{Yuk}^d D_L^c D_L^c H_L + g_{Yuk}^e E_L^c E_L^c H_L + g_{Yuk}^u U_L^c U_L^c H_L'^0 \quad (1.58)$$

From the Yukawa term (1.56a) one gets

$$\mathcal{L}_{Yuk} \supset g_{Yuk}^d \bar{d}_R d_L \phi_H + g_{Yuk}^e \bar{e}_R e_L \phi_H + g_{Yuk}^u \bar{u}_R u_L \phi_H'^0 \quad (1.59)$$

and if the neutral Higgs scalars have a vacuum expectation value (vev), then one obtains the quark and lepton masses:

$$m_d = g_{Yuk}^d \langle 0 | \phi_H | 0 \rangle, \quad m_e = g_{Yuk}^e \langle 0 | \phi_H | 0 \rangle, \quad m_u = g_{Yuk}^u \langle 0 | \phi_H' | 0 \rangle \quad (1.60)$$

However, at the same time, from the scalar Lagrangian, (155d), one gets

$$\begin{aligned} \mathcal{L}_{\text{Scalar}} \supset & g_{Yuk}^d (|\phi_{dL}\phi_{H_L^0}|^2 + |\phi_{dL}\phi_{H_L^\pm}|^2 + |\phi_{dL}\phi_{dL^\pm}|^2) \\ & + g_{Yuk}^e (|\phi_{eL}\phi_{H_L^0}|^2 + |\phi_{eL}\phi_{H_L^\pm}|^2 + |\phi_{eL}\phi_{eL^\pm}|^2) \\ & + g_{Yuk}^u (|\phi_{uL}\phi_{H_L^0}|^2 + |\phi_{uL}\phi_{H_L^\pm}|^2 + |\phi_{uL}\phi_{uL^\pm}|^2), \end{aligned} \quad (1.61)$$

from which you can obtain left and left-conjugate squark and slepton masses equal to their partner masses. This applies to all the supersymmetric particles in general. Since no supersymmetric particles have been found with the same mass as their normal particle, supersymmetry must be broken.

The squark masses given by equation (1.61) are clearly diagonalized in flavour space by the same matrices that diagonalize the quark matrix, equation (1.60), as $(S^\dagger M M^\dagger S) = (T^\dagger M^\dagger M T) = M_d^2$, where S and T are the left and right unitary transformations defined in section 1.4. If one was to add some general mass matrix to this, one would undoubtedly destroy the ability to simultaneously diagonalize both the quark and squark mass matrices. If this were so, one would be able to generate unacceptably high flavour-changing interactions through gaugino-squark loops. In order to eliminate this problem, one must postulate squark mass terms that are simultaneously diagonalizable with the quark masses. Such terms are [Ell.83]:

$$\begin{pmatrix} \phi_{qL}^* & \phi_{qL} \end{pmatrix} \begin{bmatrix} L^2 \tilde{m}^2 \mathbf{1} + C_{2L} |m_q|^2 & C_1 \tilde{m} m_q \\ C_1 \tilde{m} m_q^\dagger & R^2 \tilde{m}^2 \mathbf{1} + C_{2R} |m_q|^2 \end{bmatrix} \begin{pmatrix} \phi_{qL} \\ \phi_{qL}^* \end{pmatrix}, \quad (1.62)$$

where each square in the matrix represents a three by three matrix in flavour space diagonalized by $S^\dagger (M^2)_{11} S$, $T (M^2)_{22} T^\dagger$, $S^\dagger (M^2)_{21} T$, or $T^\dagger (M^2)_{12} S$. L , R , C_1 , C_{2L} , C_{2R} are all dimensionless constants, while $\tilde{m}$ is a constant with dimensions of mass. In some scenarios of supersymmetry breaking, $\tilde{m}$ would be approximately equal to the gravitino mass. The dimensionless constants are all of order one, and therefore one expects a certain mixing of helicity states with an amplitude:

$$\Delta m_{L,R}^2 \approx m_q \tilde{m}. \quad (1.63)$$

Such mixing can lead to contributions to $g-2$ of the electron and muon of the order of [Ell.82, Bar.82, Fay.80, Moh.86]:

$$\frac{1}{2}\Delta(g-2)_\mu = \frac{\alpha}{12\pi} \left(\frac{m_\mu}{\tilde{m}}\right)^2, \quad (1.64)$$

constraining the supersymmetric muon mass to be greater than 10 GeV. π_{2l} decay is another helicity suppressed decay which constrains the masses to be of the same order [Cam.84].

Even with the choice of equation (1.60) for tree level squark and slepton masses one will still have flavour-changing interactions introduced by one loop corrections to the masses. Contributions from loop diagrams like figure 1.7, which are allowed due to the scalar Lagrangian, equation (1.56d), lead to radiative squark masses [Hal.86]:

$$\begin{aligned} \frac{\Delta \tilde{m}_{U_L}^2}{\tilde{m}^2} &\propto K \frac{m_d^2}{M_W^2} K^\dagger, \\ \frac{\Delta \tilde{m}_{D_L}^2}{\tilde{m}^2} &\propto K^\dagger \frac{m_u^2}{M_W^2} K, \end{aligned} \quad (1.65)$$

where m_u and m_d are the real and diagonal quark matrices and K is the C-K-M matrix. Through diagrams like figure 1.8, one can then have $\Delta S = 2$ flavour-changing interactions with an amplitude:

$$M_{\text{eff}}^{\Delta S=2} = \frac{G_F \alpha}{4\pi} \frac{\Delta \tilde{m}_{Q_L}^2}{\tilde{m}^2}. \quad (1.66)$$

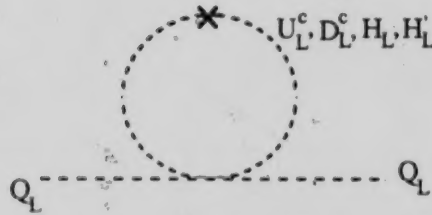


Figure 1.7: Possible radiative corrections to the left-handed squark mass. Dashed lines represent squark fields.

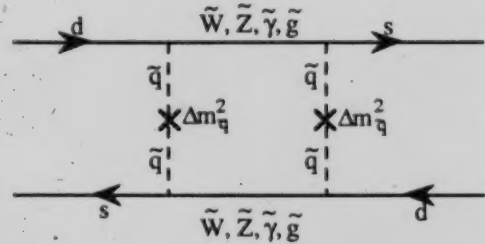


Figure 1.8: Contribution to K_L - K_S mass mixing from squark flavour mixing.

Provided the top quark mass is not too large, contributions like these would be phenomenologically acceptable even for squark masses in the 100 GeV range. In general, constraints from the K_L - K_S system constrain

$$(\text{Re}, \text{Im}) \frac{\Delta \tilde{m}_{s-d}^2}{\tilde{m}^2} \leq (10^{-4}, 10^{-7}) \left(\frac{\tilde{m}}{100 \text{ GeV}} \right)^2. \quad (1.67)$$

Constraints from the lepton sector on the mass splitting between different flavours of sleptons are even more stringent.

As one can see, flavour-changing interactions again play an important role in determining the allowable squark mass matrices, and may still play a role in restricting the masses of the squark and slepton masses for given models of mass splitting between squark and slepton flavours.

1.14 Some Supersymmetric Phenomenology

Much of the supersymmetric phenomenology relevant to this thesis was discussed in the previous section. Once again, it should be stressed that flavour-changing interactions and other rare decays, particularly, helicity suppressed decays can provide good constraints on supersymmetric models. What is left is short discussion on direct searches.

The terms in the superpotential of equation (1.59) do not represent all the gauge invariant cubic terms possible. Terms like $(L_L L_L E_L^c)$, $(L_L Q_L D_L^c)$, and $(D_L^c D_L^c U_L^c)$ are also possible and they can lead to rapid proton decay. In order to forbid such terms one must impose a symmetry known as R-parity, $R = (-1)^{3B+2S+L}$. In essence this means that all supersymmetric particles must be produced in pairs and that the lightest supersymmetric particle (LSP) is stable. Cosmological constraints on the lightest supersymmetric particle impose that this particle must be a neutral colourless particle, probably one of the neutral

gauginos, a linear combination of the photino and the other spin-1/2 neutralino particles.

The implications of this are that any produced supersymmetric particle must have a decay chain that ends with the production of the LSP. To experimentally search for supersymmetric particles, one can look for the pair production of new heavy particles whose decays carry off missing energy and momentum. One also has the advantage that the SUSY particles will be produced with the known coupling constants of the standard gauge interactions, so fairly precise calculations of cross-sections are possible, with the only unknown being the particle masses. In hadron colliders, the coloured squarks and gluinos will be copiously produced. Constraints from the CDF collaboration of Fermilab give [Fra.90] $m_{\tilde{g}} \geq 73$ GeV and $m_{\tilde{q}} \geq 74$ GeV at the 90% confidence level. Constraints on sleptons and charginos (a linear combination of the winos and charged higgsinos) are best gotten at e^+e^- colliders. Negative searches at LEP have produced the bounds, $m_{\tilde{l}} \geq 40$ GeV for the sleptons and $m_{\tilde{\chi}} \geq 45$ GeV for the charginos. Since the neutralinos are a linear combination of the neutral gauginos and neutral higgsino their cross-sections are highly model dependent. LEP searches have been able to constrain some of the parameter space on these models, but precise mass constraints are much harder, and none will be listed here.

Finally, there may be some indirect evidence for supersymmetry. If one makes the assumption that the coupling constants meet at some grand unified scale, one can use the renormalization group equations (RGE) to determine this scale and the Weinberg angle as was done in section 1.5. Since the Weinberg angle, θ_W , is an experimental quantity, one has actually over constrained the system. Recent results from the DELPHI collaboration at LEP [Ama.91] have led to the conclusion that a supersymmetric model actually gives a better match to experiment than a non-supersymmetric model does, as above the supersymmetric particle spectrum threshold, the RGE's change. The recent

evidence would support SUSY masses in the 1 TeV range.

1.15 What Lies Ahead

As I have tried to demonstrate over the course of this chapter, rare decays, and in particular, flavour-changing interactions have proven useful tools to constrain the Standard Model and some of its non-standard extensions or replacements. Flavour-changing in the K_L - K_S system allowed physicists to postulate [Gla.70] the charm quark and even get a fairly precise estimate [Gai.74, Gai.75] of its mass. Similar constraints from the B system may soon do the same for the top quark. Flavour-changing interactions have also eliminated the simplest Technicolour schemes and severely limited the squark mass matrices in supersymmetry. Finally, flavour-changing interactions have shown that composite models must have scales in the 100 TeV range. In this thesis we will see what constraints on compositeness follow from π_{2I} decays, muon conversion, K_L - K_S mass mixing and from the neutron electric dipole moment. More precisely, we will show that in composite theories with supersymmetry, rare decays will not be as suppressed as in a pure composite theory, and therefore more severe constraints will result.

Also in this thesis, we will see that electromagnetic transitions of $b \rightarrow s$ can restrict possible non-Standard Model contributions to the $WW\gamma$ vertex. Such contributions could arise in composite gauge boson models.

2. TESTS OF SUPERCOMPOSITENESS FROM RARE PROCESSES

2.1 Introduction

In the introductory chapter, it was shown that rare processes have been able to constrain various extensions to and replacements of the Standard Model. In particular, we discussed that composite models typically have effective four-fermion interactions that do not necessarily obey Standard Model conservation laws like flavour-conservation. The leading contributions from such interactions would typically be suppressed by two powers of the composite mass scale, M_C . In what follows, we will see how rare processes such as π_{21} decay, muon conversion and the K_L - K_S mass difference can constrain such composite models. More importantly, we will also consider supersymmetric composite models. These models may contain two-fermion, two-scalar interactions, that by dimensional arguments need only be scaled by one power of the composite mass scale. The two scalars can then be dressed to two fermions, further suppressing the final amplitude by one power of a scale given by the supersymmetric masses. If the scale of supersymmetric masses is significantly smaller than the scale of compositeness, then these processes will not be as suppressed as the direct four-fermion decays, and therefore will serve as a more stringent test of the composite model. Such a process should be able to appear in any theory that is both composite and supersymmetric (supercomposite), unless there is some mechanism to forbid it. The bulk of this chapter, and the next chapter, will be devoted to setting limits on supercomposite models from the above mentioned rare processes using the less severely suppressed two-scalar, two-fermion effective-interactions dressed to a four-fermion interaction as discussed above.

Before we do that, we shall first discuss a second method of obtaining small fermion masses in a composite model, the quasi-Goldstone fermion theories. These theories are supersymmetric and as such will provide motivation for our studies of

supercomposite models.

2.2 An Example of Supercompositeness - Quasi-Goldstone Fermions

If the preons have some global symmetry, G , broken to a smaller symmetry, H , then by Goldstone's theorem one has $\dim \frac{G}{H}$ massless Goldstone bosons. If the theory is supersymmetric, then one will have the same number of massless chiral fermions produced, the supersymmetric partners of the Goldstone bosons. It is these fermions, the quasi-Goldstone fermions, that one tries to associate with the quarks and leptons [Buc.83]. In addition, one also obtains $\dim \frac{G}{H}$ quasi-Goldstone bosons, a product of the supersymmetry. Actually due to some subtleties, one need only have up to $\dim \frac{G}{H}$ quasi-Goldstone fermions, and up to $\dim \frac{G}{H}$ quasi-Goldstone bosons such that the sum of Goldstone bosons and quasi-Goldstone bosons is twice the number of quasi-Goldstone fermions.

Of course, one would now wish to give mass to the Goldstone bosons so as not to conflict with experiment. One can do this by breaking supersymmetry. However, it is not so easily done as in the purely supersymmetric case. Here the massless Goldstone bosons provided the inspiration for the existence of the massless quasi-Goldstone fermions. Would not giving them mass destroy the dynamics for having low mass fermions? Obviously to preserve this dynamical reason for small fermion masses, one would require the supersymmetry breaking scale to be below the composite mass scale.

Actually, the solution to this is fairly natural. The Goldstone bosons will remain massless as long as the symmetry G remains exact. Typically, one expects that the supersymmetry breaking does not affect G , but G is explicitly broken down when the gauge interactions are turned on. Letting α stand for the square of a generic gauge coupling constant, and α_H for the coupling to a heavy sector which would appear at the composite mass scale, M_c , one has for the boson masses [Buc.83]:

$$\begin{aligned} m_{GB}^2 &= \alpha \tilde{m}^2 \ln (M_c / \tilde{m}), \quad (\text{Goldstone bosons}) \\ m_{QGB}^2 &= \alpha_H \tilde{m}^2 \ln (M_c / \tilde{m}), \quad (\text{quasi-Goldstone bosons}) \end{aligned} \quad (2.1)$$

where $\tilde{m}$ is the supersymmetry breaking scale. At the same time, the fermions acquire a mass:

$$m_{QGF} \approx \alpha_H \tilde{m} \frac{\Delta M}{M_c} \ln (M_c / \tilde{m}), \quad (2.2)$$

where ΔM represents the mass splitting between typical chiral partners in the heavy sector. Since one has no reason to expect ΔM to be small, and indeed it is likely to be of order the composite mass scale, the fermion masses will be just as heavy as the boson masses. Obviously, an addition mechanism is needed to keep the fermion masses small. Such a mechanism might be a chiral symmetry. In essence, the quarks and leptons would be protected by two mechanisms, they have low masses due to their Goldstone nature, as well as low masses due to their chiral nature [Bar.83]. However, one final observation should be made concerning the Goldstone boson and quasi-Goldstone fermion masses. In the above, supersymmetry breaking gives masses to all three of the Goldstone bosons, the quasi-Goldstone bosons and the quasi-Goldstone fermions. Even if some chiral protection mechanism was to appear in addition to this to keep the fermion masses small, one would still expect the fermion masses to be related to this supersymmetric mass scale. If one hopes to keep the fermion masses small, one would require the supersymmetry breaking scale to be far below the composite mass scale.

2.3 The Phenomenology of Supercomposite Models

As we have already mentioned, with composite models one expects that in general the dynamics of the constituent particles would manifest themselves at low energies by four-fermion effective interactions as demonstrated in figure 2.1a. This is a concept that dates back to Fermi's theory of the weak interactions. By integrating out the degrees of freedom corresponding to the constituent particles, one is left with interactions among the

composite particles that remain. These interactions do not have to be renormalizable, being the effective interactions that they are, however, they must respect the gauge invariance of the unbroken portion of the underlying theory. There is no reason why these interactions should necessarily respect the Standard Model conservation laws like flavour-conservation. As such we have an effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \frac{1}{M_c^2} \bar{\psi}\psi\bar{\psi}\psi. \quad (2.3)$$

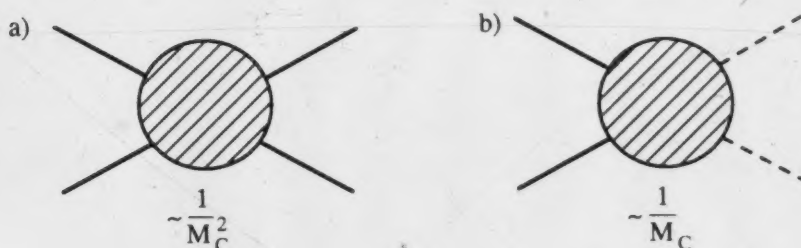


Figure 2.1 a) Four-fermion effective interaction. b) Two-fermion, two-scalar effective interaction.

Any four-fermion process would therefore have an amplitude suppressed by two powers of the composite mass scale, M_c . In a theory with an unbroken supersymmetry at the composite mass scale, one can integrate out the constituent particles leaving you with effective interactions between the constituent fermions and scalars of the supersymmetric theory. Thus one expects that the dynamics of the constituent particles might also manifest themselves at lower energies by effective two-fermion, two-scalar interactions as demonstrated in figure 2.1b with effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \frac{1}{M_c} \bar{\psi}\psi\phi^*\phi. \quad (2.4)$$

The amplitudes for these processes would only be suppressed by one power of the composite mass scale. As before these interactions must respect the gauge invariance of the unbroken underlying theory, which now, because it is supersymmetric, is not only an invariance under $SU(3)_c \otimes SU(2)_L \otimes U(1)$, but also includes invariance under the supersymmetric R-parity. Again, there is no reason to expect Standard Model

conservation rules such as flavour conservation to be respected. To obtain a four-fermion interaction, one could dress the two-scalar, two-fermion interaction with a gaugino as is illustrated in figure 2.2. Such a process would be suppressed by one power of the composite mass scale and one power of the supersymmetric mass scale, $\tilde{m}$.

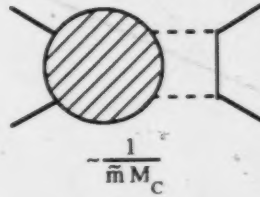


Figure 2.2 A two-fermion, two-scalar interaction dressed to a four-fermion effective interaction by the exchange of a gaugino.

We now will require a supersymmetric Lagrangian that includes these effective two-scalar, two-fermion interactions. The supersymmetric Lagrangian written down in the first chapter is not the most general supersymmetric Lagrangian since we had required it to be renormalizable. Weinberg has written down a non-renormalizable and ungauged (although it would be straight forward to do so) Lagrangian that would be applicable to our processes. It is [Wei.82]:

$$\begin{aligned}
 \mathcal{L} = & J_{ij} \partial_\mu \phi^i \partial^\mu \phi_j^* + J_{ij}^{-1} \frac{\partial f}{\partial \phi^i} \left(\frac{\partial f}{\partial \phi^j} \right)^* + J_{ij} \bar{\psi}_i \partial \psi_j \\
 & + \frac{1}{2} (\bar{\psi}_L^i)^T C^{-1} \psi_L^j \left[\frac{\partial^2 f}{\partial \phi^i \partial \phi^j} - J_{km}^{-1} \left(\frac{\partial f}{\partial \phi^k} \right) \frac{\partial^3 d}{\partial \phi_m^* \partial \phi^i \partial \phi^j} \right] \\
 & + \frac{1}{2} \bar{\psi}_L^j C (\bar{\psi}_L^i)^T \left[\left(\frac{\partial^2 f}{\partial \phi^i \partial \phi^j} \right)^* - J_{mk}^{-1} \left(\frac{\partial f}{\partial \phi^k} \right)^* \frac{\partial^3 d}{\partial \phi^m \partial \phi_i^* \partial \phi_j^*} \right] \\
 & - \frac{1}{2} \bar{\psi}_L \gamma^\mu \psi_L \left[\frac{\partial^3 d}{\partial \phi^i \partial \phi_j^* \partial \phi^m} \partial_\mu \phi^m - \frac{\partial^3 d}{\partial \phi^i \partial \phi_j^* \partial \phi_m^*} \partial_\mu \phi_m^* \right] \\
 & - \frac{1}{4} [(\bar{\psi}_L^i)^T C^{-1} \psi_L^j] [\bar{\psi}_L^k C (\bar{\psi}_L^m)^T] \left[\frac{\partial^4 d}{\partial \phi^i \partial \phi^j \partial \phi_k^* \partial \phi_m^*} + J_{np}^{-1} \frac{\partial^3 d}{\partial \phi^i \partial \phi^j \partial \phi_n^*} \frac{\partial^3 d}{\partial \phi^p \partial \phi_k^* \partial \phi_m^*} \right],
 \end{aligned} \tag{2.5}$$

where

$$J_{ij} = \frac{\partial^2 d}{\partial \Phi^i \partial \Phi_j^*}. \quad (2.6)$$

The f-term is the superpotential, that is $f=f(\Phi)$, however, it is no longer restricted to be no more than cubic in the superfield. The d-term is a function of both the superfield and its complex conjugate, $d=d(\Phi, \Phi^*)$. The last line in the Lagrangian together with a dimension-six (quartic) d-term will produce the four-fermion interaction we mentioned earlier. However, we are more interested in the second and third lines of equation (2.5), as they will produce a two-fermion, two-scalar interaction if one deals with dimension-five f-terms (quartic) or d-terms (cubic).

These f- and d-terms must respect gauge invariance and R-parity. The dimension-five f- and d-terms that are invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)$ and R-parity are [Wei.82, Sak.82]:

$$\begin{array}{lll} (L_L E_L^c H_L^*)_d & (Q_L D_L^c H_L^*)_d & (Q_L U_L^c H_L^*)_d \\ (L_L L_L H_L^c H_L)_f & (Q_L Q_L U_L^c D_L^c)_f & (Q_L L_L U_L^c E_L^c)_f \\ (Q_L Q_L Q_L L_L)_f & (U_L^c U_L^c D_L^c E_L^c)_f & \end{array} \quad (2.7)$$

The superfield definitions are as given in Chapter 1: Q_L and L_L are the quark and lepton doublets respectively, U_L^c , D_L^c , and E_L^c are the up, down and electron left-conjugate fields, and H_L and H_L^c are the Higgs doublets that give mass to the down quarks / leptons and the up quarks respectively. The gauge quantum numbers of each superfield are given in Appendix C. Since we are dealing with a composite theory here, the superpotential terms involving Higgs fields need not exist (this includes the dimension-four terms given in Chapter 1), as it may be possible to eliminate these fields from the theory.

One notices that all the dimension-five d-terms involve a Higgs field. If the terms involving Higgs fields exist, this means that they must be coupled to a dimension-four f-term involving a Higgs field in order to obtain a two-fermion, two-scalar interaction involving quarks and leptons. Since these dimension-four f-terms are just the known

Higgs couplings, these contributions are accompanied by the small Yukawa couplings for quarks and leptons. Two-fermion, two-scalar terms built from a d-term will invariably be smaller than terms built from f-terms and, therefore, we will only be interested in the f-term part:

$$\mathcal{L} \supset \frac{1}{2} (\psi_L^i)^T C^{-1} \psi_L^j \frac{\partial^2 f}{\partial \Phi^i \partial \Phi^j} + \frac{1}{2} \bar{\psi}_L^i C (\bar{\psi}_L^j)^T \left(\frac{\partial^2 f}{\partial \Phi^i \partial \Phi^j} \right)^* \quad (2.8)$$

Starting from the Lagrangian of equation (2.8) and the f-terms of equation (2.7) we will attempt to calculate amplitudes for various four-fermion rare processes by starting with a two-fermion, two-scalar interaction and dressing it with the gaugino with the largest possible coupling. This will be done in the context of a general supercomposite framework.

2.4 Constraints on Supercomposite Models from π_{2l} Decay

The first four-fermion decay we will look at breaks no Standard Model conservation rules, however, it is chirally suppressed in the Standard Model. As such, the decay of the pion into two leptons should be a good place to look for non-Standard Model contributions. Already, this decay has been used to test supersymmetric models [Cam.84, Sha.82] and charged Higgs models [Don.79, Hab.79, McW.81], as well as constrain the masses of leptoquarks [Sha.82]. The most general four-fermion interaction contributing to $\pi^+ \rightarrow e^+ \nu_e$ is

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} [(\bar{d} \gamma_\mu \gamma_5 u) (\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e) f_{AL}^e + (\bar{d} \gamma_5 u) (\bar{\nu}_e (1 + \gamma_5) e) f_{PL}^e] + \text{R.H. terms.} \quad (2.9)$$

In the standard weak theory of interactions, $f_{AL}^e = f_{AL}^\mu = 1$, and all other terms are absent. If one has a contribution to f_{PL} , it may be noticeable due to the interference between f_{AL} and f_{PL} . Because of this interference, the process is sensitive to f_{PL} and not its square. The ratio:

$$R = \frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} \quad (2.10)$$

can be written [Sha.82] as:

$$R = R_0 \left(1 + 2 \frac{m_\pi}{m_e} \frac{a_p}{a_A} f_{PL}^e \right) / \left(1 + 2 \frac{m_\pi}{m_\mu} \frac{a_p}{a_A} f_{PL}^\mu \right), \quad (2.11)$$

where R_0 is the Standard Model prediction with radiative corrections, Δ ,

$$R_0 = \frac{m_e^2}{m_\mu^2} \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} (1 + \Delta) = 1.233 \times 10^{-4}, \quad (2.12)$$

and

$$\begin{aligned} \langle 0 | \bar{d} \gamma_\mu \gamma_5 u | \pi^+(k) \rangle &= k_\mu m_\pi a_A / \sqrt{2m_\pi}, \\ \langle 0 | \bar{d} \gamma_5 u | \pi^+(k) \rangle &= m_\pi^2 a_p / \sqrt{2m_\pi}. \end{aligned} \quad (2.13)$$

The ratio of a_p to a_A can be estimated from current algebra, where one obtains

$$\frac{a_p}{a_A} = \frac{m_\pi}{(m_u + m_d)} \approx 14, \quad (2.14)$$

when $(m_u + m_d) \approx 10$ MeV.

If f_{PL}^e and f_{PL}^μ are of the same order, then one only need consider f_{PL}^e , as its coefficient is larger by an amount m_μ/m_e . One can now directly test the quantity, f_{PL}^e . The experimental value for the ratio of branching rates is $R = (1.218 \pm 0.014) \times 10^{-4}$ [PDG.90]. Using the largest allowed deviation of theory from experiment, one has

$$\frac{2m_\pi a_p}{m_e a_A} |f_{PL}^e| \leq \left| \frac{1.204}{1.233} - 1 \right| = 0.024, \quad (2.15)$$

and

$$|f_{PL}^e| \leq 3.1 \times 10^{-6}. \quad (2.16)$$

What we need to find out is whether composite and supercomposite effects can contribute to f_{PL} . One expects that composite effects would contribute an amount [Pes.81, Moh.86]

$$f_{PL}^e \approx f_{PL}^\mu \approx \frac{\sqrt{2}}{G_F M_c^2}, \quad (2.17)$$

yielding a limit on M_c of $M_c \geq 200$ TeV.

What about supercomposite theories, what contribution would they have from 2-fermion, 2-scalar interactions? Figure 2.3 shows the different possibilities for a supercomposite effect to π_{2l} decay. The two-fermion, two-scalar interaction is obtained from the superpotential,

$$f \supset \frac{1}{M_c} Q_L U_L^c L_L E_L^c, \quad (2.18)$$

from which for gluino exchange, one can obtain an amplitude for the first diagram of:

$$\begin{aligned} A_{\tilde{g}} &= i(\sqrt{2})^2 g_s^2 \frac{-1}{M_c} \int \frac{d^4 k}{(2\pi)^4} \left(\frac{\bar{d} R (k + M_{\tilde{g}}) R u}{k^2 - M_{\tilde{g}}^2} \right) (\bar{v} R e) \frac{i}{(k^2 - M_u^2)} \frac{i}{(k^2 - M_d^2)} \\ &= 2i g_s^2 \frac{\tilde{m}}{M_c} [\bar{d} R u] [\bar{v} R e] \times I, \end{aligned} \quad (2.19)$$

where

$$I = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - \tilde{m}^2)^3} = \frac{-i}{32\pi^2 \tilde{m}^2}. \quad (2.20)$$

A common supersymmetric mass has been used in the evaluation of the integral. The values for this integral and all subsequent $d^4 k$ integrals are given in Appendix D.

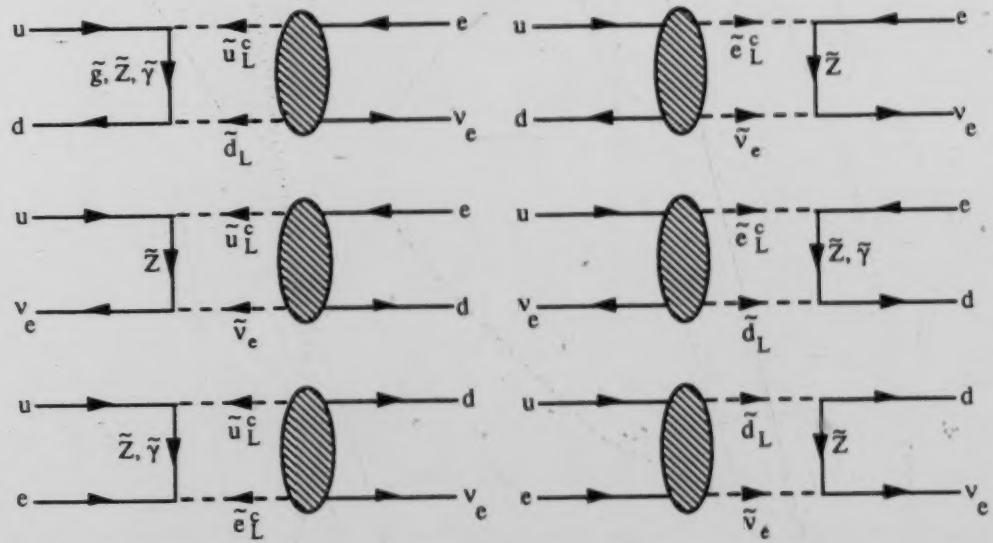


Figure 2.3: Dressed dimension-five operators leading to π_{2I} decay, only the first diagram allows the exchange of a gluino.

For completeness, we will also give the amplitude due to zino and photino exchange. The coupling of fermions to the Z, is given by:

$$\mathcal{L}_Z = \frac{ig}{\cos \theta_W} (c_L \bar{f}_L \gamma^\mu f_L + c_R \bar{f}_R \gamma^\mu f_R) Z_\mu. \quad (2.21)$$

with $c_L^f = \pm \frac{1}{2} - q \sin^2 \theta_w$ and $c_R^f = -q \sin^2 \theta_w$, the positive sign on c_L applying to isospin +1/2 states (up quarks and neutrinos) and the negative sign to isospin -1/2 states (down quarks and electrons). Thus the amplitude for the exchange of a zino in the first diagram of figure 2.3 is

$$A_{\tilde{Z}} = i(\sqrt{2})^2 c_L^d c_R^u \frac{g^2}{\cos^2 \theta_w} \frac{1}{M_c} \int \frac{d^4 k}{(2\pi)^4} \left(\frac{\bar{d}R(k+M_{\tilde{Z}})Ru}{k^2 - M_{\tilde{Z}}^2} \right) \frac{i}{(k^2 - M_d^2)} \frac{i}{(k^2 - M_u^2)} \quad (2.22)$$

$$= \frac{c_L^d c_R^u}{16\pi^2} \frac{g^2}{\cos^2 \theta_w} \frac{1}{\tilde{m}M_c} [\bar{d}Ru] [\bar{v}Re],$$

where once again a common supersymmetric mass has been used. Taking all 6 Zino exchange diagrams into consideration, one has a total amplitude

$$A_{\tilde{Z}} = \frac{C_{\tilde{Z}}}{16\pi^2} \frac{g^2}{\cos^2 \theta_w} \frac{1}{\tilde{m}M_c} (\bar{d}Ru)(\bar{v}Re), \quad (2.23)$$

where

$$C_{\tilde{Z}} = c_L^d c_R^u + c_L^u c_R^d - \frac{1}{2} c_L^d c_R^d - \frac{1}{2} c_L^u c_R^u + \frac{1}{2} c_R^d c_R^d + \frac{1}{2} c_L^u c_L^u$$

$$= -\frac{1}{8} + \frac{4}{3} \sin^2 \theta_w - \frac{13}{18} \sin^4 \theta_w$$

$$= 0.143. \quad (2.24)$$

The other diagrams were calculated in much the same manner as the first one, however, the last four must be Fierz reordered using

$$(\bar{d}Re)(\bar{v}Ru) = -\frac{1}{2}(\bar{d}Ru)(\bar{v}Re) - \frac{1}{8}(d\sigma_{\mu\nu}u)(\bar{v}\sigma^{\mu\nu}Re)$$

$$= -\frac{1}{2}(\bar{d}Ru)(\bar{v}Re), \quad (2.25)$$

as $\langle \pi | d\sigma_{\mu\nu} | 0 \rangle = 0$, $\sigma_{\mu\nu}$ being a spin-two operator while the pion is spin-zero. For photino exchange the first diagram gives an amplitude:

$$A_{\tilde{\gamma}} = i \frac{q_u q_d (\sqrt{2})^2}{M} \int \frac{d^4 k}{(2\pi)^4} \left(\frac{\bar{d}R(k+M_{\tilde{\gamma}})Ru}{k^2 - M_{\tilde{\gamma}}^2} \right) (\bar{v}Re) \frac{1}{(k^2 - M_d^2)} \frac{1}{(k^2 - M_u^2)} \quad (2.26)$$

$$= \frac{q_u q_d}{16\pi^2} \frac{1}{\tilde{m}M_c} (\bar{d}Re)(\bar{v}Re).$$

The total amplitude due to photino exchange is then

$$A = \frac{e^2 C_{\tilde{\gamma}}}{16\pi^2} \frac{1}{\tilde{m}M_c} (\bar{d}Ru)(\bar{v}Re), \quad (2.27)$$

where

$$C_{\tilde{\gamma}} = (q^d q^u - \frac{1}{2} q^d q^e + \frac{1}{2} q^e q^u) / e^2 = -\frac{13}{18}. \quad (2.28)$$

The total contribution to π_{2l} decay is now

$$A = \frac{1}{(4\pi)^2} \frac{1}{\tilde{m}M_c} \left(g_s^2 + \frac{g^2 C_Z}{\cos^2 \theta_W} + e^2 C_Y \right) (\bar{d}Ru)(\bar{\nu}Re) \quad (2.29)$$

$$= \frac{1}{4} \left(\frac{g_s}{4\pi} \right)^2 \frac{1}{\tilde{m}M_c} [\bar{d}\gamma_5 u][\bar{\nu}(1+\gamma_5)e],$$

since $\langle \pi | \bar{d}u | 0 \rangle = 0$ as the pion is a pseudoscalar particle and the zino and photino contributions have been ignored in the final analysis due to the relative strengths of the strong, weak and electromagnetic couplings. Comparing this with equation (2.9) one has that

$$f_{PL}^e = f_{PL}^\mu = \frac{1}{4} \left(\frac{g_s}{4\pi} \right)^2 \frac{\sqrt{2}}{G_F \tilde{m}M_c}, \quad (2.30)$$

which for $g_s/4\pi \approx 0.1$, one gets from the experimental limit on f_{PL}^e a constraint

$$\tilde{m}M_c \geq 98 \text{ (TeV)}^2. \quad (2.31)$$

For supersymmetric masses of order 100 GeV (1 TeV), one has that $M_c \geq 980$ TeV (98 TeV). These limits are not significantly greater than limits from the purely composite processes, as one has the additional presence of the strong coupling constant in the supercomposite processes. The supercomposite processes will start to dominate for

$$M_c \geq \frac{4\tilde{m}}{(g_s/4\pi)^2} \approx 400\tilde{m}. \quad (2.32)$$

The current constraints on supercomposite models from π_{2l} decay are not substantially greater than constraints on pure composite models from the same process. To obtain these significant gains from a supercomposite system, one needs larger constraints on the composite mass scale. Such is the case if one is considering composite models that do not conserve flavour, and thus we will look at such flavour-changing interactions in the next two sections.

2.5 Test of Supercompositeness from Muon Conversion.

Since we require a four-fermion interaction to take full advantage of the two-scalar, two-fermion interaction available in supercomposite models we are severely limited in the flavour-changing processes we can look at. The most obvious choice would

be $\mu \rightarrow 3e$, however, there is no f term involving four leptons from which to create a two-scalar, two fermion interaction that would lead to this process. The next best place to look would be muon conversion ($\mu N \rightarrow eN$). In this case there is a suitable f -term, $f \supset \frac{1}{M_c} Q_L U_L^c L_L E_L^c$, which can be dressed to the final four-fermion interaction.

The $\mu N \rightarrow eN$ effective four-fermi interaction can be parametrized by the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=0,1} g_S^{(i)} \bar{e} (1 - \alpha_S^i \gamma_5) \mu S^{(i)} + g_P^{(i)} \bar{e} (1 - \alpha_P^i \gamma_5) \mu P^{(i)} \right], \quad (2.33)$$

with quark currents

$$\begin{aligned} S^{(0)} &= \frac{1}{2} (\bar{u}u + \bar{d}d), \\ S^{(1)} &= \frac{1}{2} (\bar{u}u - \bar{d}d), \end{aligned} \quad (2.34)$$

and

$$\begin{aligned} P^{(0)} &= \frac{1}{2} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d), \\ P^{(1)} &= \frac{1}{2} (\bar{u}\gamma_5 u - \bar{d}\gamma_5 d). \end{aligned} \quad (2.35)$$

It is expected that the coherent process (i.e. the scalar quark currents) will dominate, and thus we shall ignore the pseudoscalar quark currents. For the coherent process [Sha.79]:

$$\begin{aligned} \Gamma(\mu N \rightarrow eN) &= \frac{G_F^2 E_e^2}{16\pi^2} \left\{ \left[\left(\frac{3g_S^{(0)} + g_S^{(1)}}{2} \right) ZR_Z + \left(\frac{3g_S^{(0)} - g_S^{(1)}}{2} \right) NR_N \right]^2 \right. \\ &\quad \left. + \left[\left(\frac{3\alpha_S^0 g_S^{(0)} + \alpha_S^1 g_S^{(1)}}{2} \right) ZR_Z + \left(\frac{3\alpha_S^0 g_S^{(0)} - \alpha_S^1 g_S^{(1)}}{2} \right) NR_N \right]^2 \right\}, \end{aligned} \quad (2.36)$$

where

$$R_{Z(N)} = \int (g_e g_\mu - f_e f_\mu) \rho_{Z(N)} 4\pi r^2 dr, \quad (2.37)$$

and ρ_Z, ρ_N are the proton and neutron densities (normalized to 1), g_μ, f_μ are the top and bottom components of the 1s muon wave function, g_e, f_e are the top and bottom components of the Coulomb-modified spherical electron wave function [Sha.79 and references therein]. If one assumes that the neutron and proton densities are very similar

($\rho_Z \approx \rho_N \approx \rho$), then one can simply write the branching ratio, for $\alpha_S^0 = \alpha_S^1 = \alpha$, as

$$R = \frac{\Gamma(\mu N \rightarrow eN)}{\Gamma(\mu N \rightarrow \text{all})} \approx \frac{\Gamma(\mu N \rightarrow eN)}{\Gamma(\mu N \rightarrow \nu N')} = \frac{\Gamma_G(Z)}{\Gamma(\mu N \rightarrow \nu N')} \left\{ \frac{1}{2} (1 + \alpha^2) \left[g_S^{(0)} + g_S^{(1)} \frac{(Z-N)}{3A} \right]^2 \right\}, \quad (2.38)$$

where

$$\Gamma_G(Z) = 9E_e^2 A^2 \frac{G_F^2}{2} \left[\int (g_e g_\mu - f_e f_\mu) \rho r^2 dr \right]^2. \quad (2.39)$$

The ratio $\Gamma_G(Z)/\Gamma(\mu N \rightarrow \nu N')$ has been catalogued by Shankar [Sha.79].

It is expected that the $\mu N \rightarrow e N$ process will either produce right- or left-handed electrons, or it will not depend on the handedness at all. Thus we will only be interested in the cases $\alpha = \pm 1$ and 0 in which case the branching ratio further reduces to

$$R = \left(\frac{1}{2}\right) \frac{\Gamma_G(Z)}{\Gamma(\mu N \rightarrow \nu N')} \left[g_S^{(0)} + g_S^{(1)} \frac{(Z-N)}{3A} \right]^2, \quad (2.40)$$

and the 1/2 applies for the $\alpha = 0$ case. We will be further interested in 4 cases: $g_S^{(0)} = g_S$, $g_S^{(1)} = 0$ (up and down quarks contribute equally); $g_S^{(1)} = g_S$, $g_S^{(0)} = 0$ (up and down quarks contribute equal but opposite); $g_S^{(0)} = g_S^{(1)} = g_S^{(u)}$ (only up quarks contribute); $g_S^{(0)} = -g_S^{(1)} = g_S^{(d)}$ (only down quarks contribute). In these cases

$$R = \left(\frac{1}{2}\right) (g_S^{(i)})^2 \frac{\Gamma_G(Z)}{\Gamma(\mu N \rightarrow \nu N')} \left\{ \begin{array}{ll} 1 & g_S^{(0)}, g_S^{(1)}=0, \\ \left(\frac{Z-N}{3A}\right)^2 & g_S^{(1)}, g_S^{(0)}=0, \\ \frac{2 \cdot (2Z+N)}{3A} & g_S^{(0)} = g_S^{(1)} = g_S^{(u)}, \\ \frac{2 \cdot (Z+2N)}{3A} & g_S^{(0)} = -g_S^{(1)} = g_S^{(d)}. \end{array} \right. \quad (2.41)$$

again the 1/2 applies for the $\alpha = 0$ case.

The best experimental results for the muon conversion process come from the Titanium system [PDG.90]:

$$R = \frac{\Gamma(\mu \text{-Ti} \rightarrow e \text{-Ti})}{\Gamma(\mu \text{-Ti} \rightarrow \text{all})} \leq 4.6 \times 10^{-12}. \quad (2.42)$$

The value for $\Gamma_G(22)/\Gamma(\mu \text{-Ti} \rightarrow \nu \text{Sc})$ for Ti ($Z=22$, $N=26$, $A=48$) is 228, from which the following constraints can be obtained:

$$\begin{aligned}
g_S^{(0)} &\leq 2.0 \times 10^{-7} & (1.4 \times 10^{-7}), \\
g_S^{(1)} &\leq 7.2 \times 10^{-7} & (5.1 \times 10^{-6}), \\
g_S^{(u)} &\leq 2.1 \times 10^{-7} & (1.5 \times 10^{-7}), \\
g_S^{(d)} &\leq 2.0 \times 10^{-7} & (1.4 \times 10^{-7}),
\end{aligned}
\tag{2.43}$$

where this time the unbracketed values are for $\alpha = 0$, and the bracketed values for $\alpha = \pm 1$.

For a composite model, one expects up and down quark contributions to be similar or equal and we do not expect any chiral dependence. One therefore expects to have a contribution [Pes.81, Moh.86]

$$g_S^{(0)}(\alpha = 0) = \frac{1}{M_c^2 G_F \sqrt{2}}. \tag{2.44}$$

From the experimental constraints on $g_S^{(0)}$ for $\alpha = 0$ one can impose $M_c \geq 650$ TeV.

Although one cannot obtain the decay $\mu \rightarrow 3e$ from a two-scalar, two-fermion interaction, there is no reason why it cannot be obtained from a direct four-fermion interaction. Using a general effective Lagrangian [Che.77] one obtains a constraint on the scale of compositeness of the same order as that obtained from muon conversion.

We are now ready to look for constraints on supercomposite mass scales obtained from two-scalar, two-fermion interactions. We can start by looking at the up quark contribution to this process as demonstrated in figure 2.4. The exchanged gaugino could be a gluino, zino, or photino. Since the coupling to the gluino (the strong coupling) is larger than the couplings to the photino and zino, we shall only investigate the exchange of a gluino and therefore need only consider the first of the six diagrams. The f-term that leads to the composite portion of the diagram is $f \supset \frac{1}{M_c} Q_L U_L^c L_L E_L^c$ from which one can obtain outgoing right-handed electrons, while its complex conjugate will furnish outgoing left-handed electrons.

Computing an amplitude for this first diagram of figure 2.4, one obtains the result:

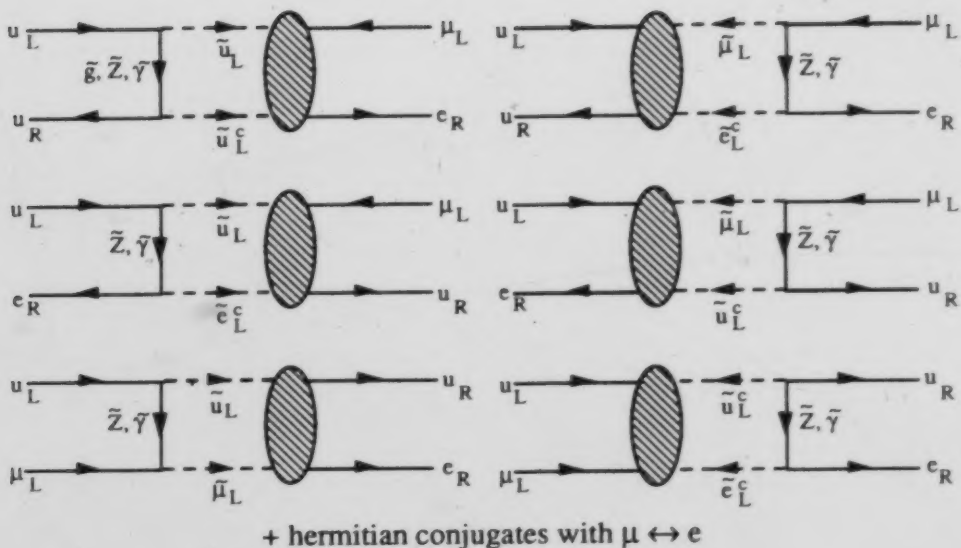


Figure 2.4 Dressed dimension-five operators leading to $\mu N \rightarrow e N$. We will only consider the first diagram.

$$\begin{aligned}
 A &= \frac{2ig_s^2}{M_c} \int \frac{d^4k}{(2\pi)^4} \left[\left(\frac{\bar{u}_L(K+M_{\tilde{g}})Lu}{k^2 - M_{\tilde{g}}^2} \right) (\bar{e}L\mu) + \left(\frac{\bar{u}_R(K+M_{\tilde{g}})Ru}{k^2 - M_{\tilde{g}}^2} \right) (\bar{e}R\mu) \right] \frac{1}{(k^2 - M_{\tilde{u}}^2)^2} \\
 &= 2i g_s^2 \frac{\tilde{m}}{M_c} [(\bar{u}Lu)(\bar{e}L\mu) + (\bar{u}Ru)(\bar{e}R\mu)] \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - \tilde{m}^2)^3} \\
 &= \frac{g_s^2}{16\pi^2} \frac{1}{\tilde{m}M_c} [(\bar{u}Lu)(\bar{e}L\mu) + (\bar{u}Ru)(\bar{e}R\mu)] \\
 &= \frac{1}{2} \left(\frac{g_s}{4\pi} \right)^2 \frac{1}{\tilde{m}M_c} [(\bar{u}u)(\bar{e}\mu) + (\bar{u}\gamma_5 u)(\bar{e}\gamma_5 \mu)].
 \end{aligned} \tag{2.45}$$

Once again we have used a common supersymmetric mass and the evaluation of the integral can be found in Appendix D.

The down quark amplitude for this process should also be calculated. However, there is no available f -term for supercomposite processes like those in figure 2.4 with the up squarks replaced with a down squarks. One could consider exchanging a charged gaugino in figure 2.4 so that a down quark is produced in the dressed process. However, this carries the much smaller weak coupling and is furthermore chirally suppressed due to the lack of a weak coupling to the conjugate-left up quark. The up quark amplitude,

therefore, represents the only relevant supercomposite contribution.

Ignoring the pseudoscalar portion of this amplitude, since the coherent (scalar) portion is the dominant contribution, and comparing equations (2.45) and (2.33) one has that

$$g_S^{(0)} = g_S^{(1)} = g_S^{(u)} = \frac{\sqrt{2}}{G_F} \left(\frac{g_s}{4\pi} \right)^2 \frac{1}{\tilde{m}M_c}, \quad \alpha_S = 0. \quad (2.46)$$

From the constraints on $g_S^{(u)}$ for $\alpha = 0$, one has for $g_s/4\pi \approx 0.1$, that

$$\tilde{m}M_c \geq 6,000 \text{ (TeV)}^2, \quad (2.47)$$

which for $\tilde{m}$ of order 100 GeV (1 TeV), one has a composite mass scale of order 6×10^4 TeV (6,000 TeV), limits considerably greater than the pure composite limit.

2.6 Tests of Supercompositeness from the K_L - K_S and B - $\bar{B}$ Systems

So far we have ignored one of the standard processes used to test exotic models, namely the K_L - K_S mass difference. This is mainly due to the fact that there is no f-term that can produce this process with the exchange of a neutral gaugino. The only f-term available is $f \supset \frac{1}{M_c} Q_L Q_L U_L^c D_L^c = \frac{1}{M_c} (U_L D_L - D_L U_L) U_L^c D_L^c$, which can only be dressed to the form $(\bar{d}\gamma_5 s)(\bar{d}\gamma_5 s)$ with the exchange of a charged gaugino. These charged "inos" are a mixture of the wino, $\tilde{W}^\pm$, and Higgsino, $\tilde{H}_L$ and $\tilde{H}_L'^+$, states:

$$\begin{aligned} \tilde{\chi}_1^+ &= \tilde{W}^+ \cos \theta_+ - \tilde{H}_L'^+ \sin \theta_+, & \tilde{\chi}_2^+ &= \tilde{W}^+ \sin \theta_+ + \tilde{H}_L'^+ \cos \theta_+, \\ \tilde{\chi}_1^- &= \tilde{W}^- \cos \theta_- - \tilde{H}_L^- \sin \theta_-, & \tilde{\chi}_2^- &= \tilde{W}^- \sin \theta_- + \tilde{H}_L^- \cos \theta_-, \end{aligned} \quad (2.48)$$

where $\tan 2\theta_\pm$ is a complicated function of supersymmetric mass terms and the Higgs VEVs. The exact detail of this dependence is not important to what follows, and, therefore, I refer the reader to references [Ros.85, Hab.85] for further details. Since we are now limited to weak decays, these supercomposite processes will be suppressed by two powers of the weak coupling constant. Furthermore, they will be chirally suppressed, leading to another suppression by the ratio of a quark mass to the weak mass.

We will be wishing to calculate the K_L - K_S mass difference, Δm , which is given by

$$\Delta m = 2 \text{Re} \langle K | \mathcal{B}_{\text{eff}} | \bar{K} \rangle. \quad (2.49)$$

Furthermore, we will be studying effective Lagrangians of the sort:

$$\mathcal{L}_{\text{eff}} = \kappa_S \frac{G_F}{\sqrt{2}} [\bar{d}R_s][\bar{d}L_s] \text{ or } \mathcal{L}_{\text{eff}} = \kappa_S \frac{G_F}{\sqrt{2}} [\bar{d}L_s][\bar{d}R_s]. \quad (2.50)$$

One can evaluate $\langle K | \mathcal{L}_{\text{eff}} | \bar{K} \rangle$ using the vacuum saturation approximation [Gai.74, McW.80] with

$$\langle 0 | \bar{d} \gamma_\mu \gamma_5 s | K \rangle = \frac{i p_\mu f_K}{\sqrt{2} m_K}, \quad \langle 0 | \bar{d} \gamma_5 s | K \rangle = \frac{i m_K \rho_P^K f_K}{\sqrt{2} m_K}, \quad (2.51)$$

and

$$\rho_P^K = \frac{m_K}{m_d + m_s} = 3.6, \quad (2.52)$$

where $m_d (= 7 \text{ MeV})$ and $m_s (= 130 \text{ MeV})$ are current quark masses. Using the vacuum saturation approximation one has that

$$\begin{aligned} \langle K | [\bar{d}R_s][\bar{d}L_s] | \bar{K} \rangle &= \langle K | [\bar{d}L_s][\bar{d}R_s] | \bar{K} \rangle \\ &= \frac{1}{4} B_B \left[\langle K | [\bar{d} (1 + \gamma_5) s] | 0 \rangle \langle 0 | [\bar{d} (1 - \gamma_5) s] | \bar{K} \rangle \right. \\ &\quad \left. + (-\frac{1}{2})(\frac{1}{3}) \langle K | [\bar{d} \gamma_\mu (1 - \gamma_5) s] | 0 \rangle \langle 0 | [\bar{d} \gamma_\mu (1 + \gamma_5) s] | \bar{K} \rangle \right] \\ &= \frac{1}{4} B_B \left[-\langle K | [\bar{d} \gamma_5 s] | 0 \rangle \langle 0 | [\bar{d} \gamma_5 s] | \bar{K} \rangle + (\frac{1}{2})(\frac{1}{3}) \langle K | [\bar{d} \gamma_\mu \gamma_5 s] | 0 \rangle \langle 0 | [\bar{d} \gamma_\mu \gamma_5 s] | \bar{K} \rangle \right] \\ &= -\frac{1}{4} \frac{f_K^2 m_K^2}{2 m_K} B_B \left[(\rho_P^K)^2 - \frac{1}{6} \right]. \end{aligned} \quad (2.53)$$

The $-1/2$ and $1/3$ are introduced in the second line above since the Fierz transform (Appendix E) of $R \otimes L = -1/2 (\gamma_\mu L \otimes \gamma_\mu R)$ and there is a $1/3$ chance of matching colours after transforming. Going from line two to three is just indicative of the fact that the kaon is a pseudoscalar particle. The B_B is inserted to account for discrepancies from the vacuum saturation model. If a value for B_B is needed, we shall choose it equal to unity. The mass difference due to the effective Lagrangian above is therefore

$$\Delta m = \frac{-\kappa_S G_F^2 f_K^2 m_K B_B}{4 \sqrt{2}} \left[(\rho_P^K)^2 - \frac{1}{6} \right]. \quad (2.54)$$

One expects that any non-Standard Model contributions to the mass difference should be smaller than the contribution to the mass difference from the Standard Model. In the Standard Model, one has a $K_L - K_S$ mass difference

$$\Delta m_{\text{SM}} = \frac{G_F^2 f_K^2 m_K B_B}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \frac{m_c^2 - m_u^2}{M_W^2 \sin^2 \theta_W} \sin^2 \theta_c \cos^2 \theta_c, \quad (2.55)$$

where θ_W and θ_c are the Weinberg and Cabbibo angles respectively. Imposing that non-

Standard Model contributions must be less than standard model contributions, one has that

$$|\kappa_S| \leq \frac{\alpha_{em}}{\pi} \frac{m_c^2 - m_u^2}{M_W^2 \sin^2 \theta_W} \frac{\sin^2 \theta_c \cos^2 \theta_c}{[(\rho_P^K)^2 - \frac{1}{6}]} = 2.0 \times 10^{-8}. \quad (2.56)$$

Once again we will first calculate possible contributions from direct four-fermion interactions in composite models. Here one expects an effective Lagrangian [Pes.81, Moh.86]

$$\mathcal{L}_{eff} = \frac{1}{M_C^2} [\bar{d}_L s_R][\bar{d}_R s_L], \quad (2.57)$$

which implies that

$$|\kappa_S| = \frac{1}{M_C^2} \frac{\sqrt{2}}{G_F}, \quad (2.58)$$

setting a constraint on the composite mass scale of 2500 TeV, a rather imposing constraint.

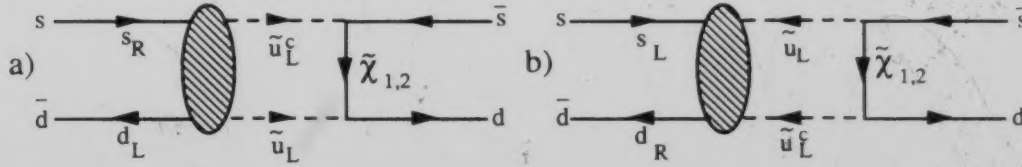


Figure 2.5: Dressed dimension-five operator contributing to the K_L - K_S mass difference.

What kind of constraints can we expect from two-scalar, two-fermion supercomposite interactions dressed to four-fermion interactions with a charged gaugino. Through an f-term, $f \supset \frac{1}{M_c} Q_L Q_L U_L^c D_L^c$, one can produce interactions like those displayed in figure 2.5.

In order to evaluate the amplitude we will require the interaction Lagrangians for the charginos with matter. This Lagrangian is:

$$\begin{aligned} \mathcal{L}_{int} = & \tilde{u}_L^* [\tilde{\chi}_1 (g \cos \theta_L L - g^d \sin \theta_R R) K d] + \tilde{u}_L^* [\tilde{\chi}_2 (g \sin \theta_L L - g^d \cos \theta_R R) K d] \\ & + g^u \sin \theta_+ \tilde{u}_L^* [(\tilde{\chi}_1^*)^T (-C^{-1}) K L d] + g^u \cos \theta_+ \tilde{u}_L^* [(\tilde{\chi}_2^*)^T (-C^{-1}) K L d] + h.c. \end{aligned} \quad (2.59)$$

R and L are the usual left and right projection operators, C is the charge conjugation

matrix, and K is the Kobayashi-Maskawa matrix. g^u and g^d are up and down Yukawa couplings respectively, with values

$$g^u = \frac{gm_u}{M_W \sqrt{2} \cos \alpha}, \quad g^d = \frac{gm_d}{M_W \sqrt{2} \sin \alpha}, \quad (2.60)$$

where $\tan \alpha = v/v'$ is the ratio of Higgs VEVs. Furthermore, we will also require the effective Lagrangian for the supercomposite portion, which using equation (2.8) is:

$$\mathcal{L}_{\text{int}} = -\frac{V_{ijkl}}{M_c} [\bar{d}_R^j] \tilde{u}_L^i \tilde{u}_L^l - \frac{V_{ijkl}^*}{M_c} [\bar{d}_L^j] (\tilde{u}_L^i)^* (\tilde{u}_L^l)^*, \quad (2.61)$$

where the i, j, k , and l are generation indices. V_{ijkl} is unknown, but typically $O(\pm 1)$. From this point onward we will suppress the generation coefficient V_{ijkl} until the final stage where we will consider the possibility that the various contributions add together, or subtract from one another.

Rather than do an in depth calculation involving the exchange of both $\tilde{\chi}_1$ and $\tilde{\chi}_2$ it is convenient to observe how the amplitudes will depend on the chargino mixing parameters. In figure two, the up squark lines can be either up, charm, or top squarks, with the appropriate C-K-M matrix element coupling them to the s or d quark. Evaluating the amplitude for figure 2.5a with the exchange of a $\tilde{\chi}_1$ will yield a result proportional to

$$A \propto -f(m_1) g^u g^d \sin \theta_+ \cos \theta_- [\bar{d}(\not{p}_s + \not{p}_d) L s] + h(m_1) g^u g^d \sin \theta_+ \sin \theta_- [\bar{d} L s], \quad (2.62)$$

where f and h are functions of the loop masses, and in particular, the mass of the gauginos. The exchange of $\tilde{\chi}_2$ leads to an amplitude proportional to

$$A \propto f(m_2) g^u g^d \cos \theta_+ \sin \theta_- [\bar{d}(\not{p}_s + \not{p}_d) L s] + h(m_2) g^u g^d \cos \theta_+ \cos \theta_- [\bar{d} L s]. \quad (2.63)$$

One can observe two things here. Firstly, since the weak coupling is stronger than the Yukawa coupling (at least for light quarks), one would expect the first term in each of these amplitudes to dominate. However, this term is also accompanied by external momentum dependence with $\bar{d}(\not{p}_s + \not{p}_d) \gamma_5 s \approx m_K \bar{d} \gamma_5 s$. The m_K suppression therefore puts this term on par with the second term with the two Yukawa couplings. The second point is what happens to the sum of these two amplitudes when $m_1 = m_2$ and $\theta_+ = \theta_-$:

$$\begin{aligned} -f(m_1)gg^u \sin \theta_+ \cos \theta_- + f(m_2)gg^u \cos \theta_+ \sin \theta_- &= 0, \\ h(m_1)g^u g^d \sin \theta_+ \sin \theta_- + h(m_2)g^u g^d \cos \theta_+ \cos \theta_- &= h(m_{\tilde{\chi}})g_1 g_2. \end{aligned} \quad (2.64)$$

Although this may or may not be a reasonable assumption (for instance $\theta_+ = \theta_-$ requires the two Higgs VEVs to be equal), it does show that we have little to gain by picking an arbitrary θ_+, θ_- . Rather it will be much more convenient to simplify, choosing $\theta_+ = \theta_- = 0$, leaving $\tilde{\chi}_2$, now a pure Higgsino state, as the only available gauge fermion to dress the interaction.

In this now simplified form, the amplitude for figure 2.5a is

$$\begin{aligned} A &= \frac{[d_L s_R]}{M_c} \int \frac{d^4 k}{(2\pi)^4} \frac{K_{u^c s} K_{ud}^*}{(k^2 - m_{\tilde{u}_L}^2)(k^2 - m_{\tilde{u}_L}^2)} \frac{g^{u^c} g^d}{(k^2 - m_{\tilde{\chi}}^2)} m_{\tilde{\chi}} [\bar{d} L s] \\ &= \frac{-i}{32\pi^2} \frac{K_{u^c s} K_{ud}^*}{\tilde{m} M_c} g^u g^d [\bar{d} R s][\bar{d} L s], \end{aligned} \quad (2.65)$$

where once again a common supersymmetric mass, $\tilde{m}$, has been used. In Appendix D, one can find an evaluation of this integral where the gaugino mass could be different than the squark masses. The u^c label in the C-K-M matrix and Yukawa couplings signifies that the label (either u, c, or t) should be identified with the flavour of the $\tilde{u}_L$ squark line. For figure 2.5b one gets a similar amplitude:

$$A = \frac{-i}{32\pi^2} \frac{K_{us} K_{u^c d}^*}{\tilde{m} M_c} g^{u^c} g^s [\bar{d} L s][\bar{d} R s]. \quad (2.66)$$

The effective Lagrangian can now be written down. It is:

$$\mathcal{L}_{\text{eff}} = \frac{-1}{32\pi^2} \frac{1}{\tilde{m} M_c} \sum_{u, u^c} \{ g^{u^c} g^d K_{u^c s} K_{ud}^* [\bar{d} R s][\bar{d} L s] + g^{u^c} g^s K_{us} K_{u^c d}^* [\bar{d} L s][\bar{d} R s] \}. \quad (2.67)$$

Comparing equations (2.67) to (2.50) and inserting the values of g^u and g^d (equation (2.60)) with $\tan \alpha = v/v' = 1$ so that $\sqrt{2} \cos \alpha = \sqrt{2} \sin \alpha = 1$ we find

$$\kappa_s = \frac{1}{4\pi^2} \frac{1}{\tilde{m} M_c} \sum_{u, u^c} \{ m_{u^c} m_d K_{u^c s} K_{ud}^* + m_{u^c} m_s K_{us} K_{u^c d}^* \}. \quad (2.68)$$

The main contributions to this will come from three possibilities: The first term when $\tilde{u}_L$ is a charm squark, $\tilde{u}_L$ an up quark; the second term when $\tilde{u}_L$ is a charm squark, $\tilde{u}_L$ an up squark, or $\tilde{u}_L$ is a charm squark and $\tilde{u}_L$ is also a charm squark. As was mentioned earlier, the relative sign between each of these terms is unknown, so we will consider two

solutions, one when all the terms add, the second when the two smaller terms subtract from the largest. Doing that we obtain:

$$\kappa_s = \frac{1}{4\pi^2} \frac{1}{\tilde{m}M_c} [m_c m_d \cos^2 \theta_c \pm (m_u m_s \cos^2 \theta_c + m_c m_s \sin \theta_c \cos \theta_c)] = \frac{1}{4\pi^2} \frac{m_q^2}{\tilde{m}M_c}, \quad 2.69$$

with $0.2 (\text{GeV})^2 \leq m_q^2 \leq 0.9 (\text{GeV})^2$. From this one can set constraints on $\tilde{m}M_c$ of:

$$\begin{aligned} \tilde{m}M_c &\geq 1.1 \times 10^6 (\text{GeV})^2, \quad m_q^2 = 0.9 (\text{GeV})^2, \\ &\text{or} \\ \tilde{m}M_c &\geq 2.5 \times 10^5 (\text{GeV})^2, \quad m_q^2 = 0.2 (\text{GeV})^2. \end{aligned} \quad (2.70)$$

Assuming $\tilde{m}$ is of the order of 100 GeV (or it is equal to the composite scale), we have constraints on the composite mass scale of 11 TeV (1.1 TeV) and 3 TeV (0.5 TeV).

These limits are far below the limits that can be obtained through a pure composite four-fermion interaction, mainly due to the two powers of the weak coupling constant and the two quarks masses required due to the chiral constraints. Indeed this process might not occur at all if some other mechanism besides Higgs Yukawa couplings are responsible for the quark masses. The supercomposite process would start to dominate over the pure composite process if

$$M_c \geq \frac{4\pi^2 \tilde{m}}{m_q^2 G_F \sqrt{2}} = \begin{aligned} &5 \times 10^6 \tilde{m}, \quad m_q^2 = 0.9 (\text{GeV})^2 \\ &24 \times 10^6 \tilde{m}, \quad m_q^2 = 0.2 (\text{GeV})^2 \end{aligned}, \quad (2.71)$$

a constraint one is not likely to achieve in the near future.

One can now ask if there is a way to get around this light quark suppression. By going to the $B-\bar{B}$ system one gets rid of one of the light quarks, since now $m_q^2 = m_t m_d$. However, one can do better still. Our problem is that we cannot couple the right-handed (left-handed conjugate) squarks to the wino. This can only be done if its chirality is flipped. In general, left- and right-handed squarks mix with an amplitude, $\Delta \tilde{m}_{L-R} \approx \tilde{m} m_q$, m_q being the mass of the associated quark. If it is the chirality of a top squark which we are flipping, as in the $B-\bar{B}$ system, we could make significant gains, provided the supersymmetric mass scale is of the same order as the weak scale.

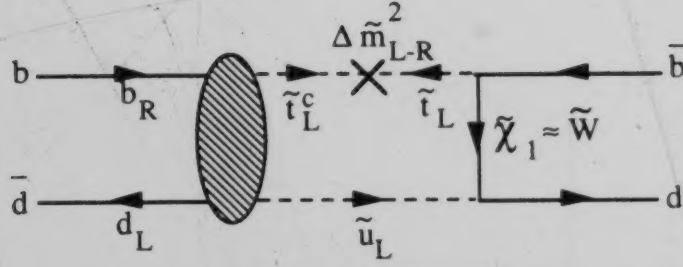


Figure 2.6: Dressed dimension-five operator contributing to $B-\bar{B}$ mass difference with a chiral flip on the squark line so that the exchange of a $\tilde{\chi}_1$ (primarily a wino, $\tilde{W}$) is allowed.

Let's consider figure 2.6, where now the dominant contribution comes from the chargino that is predominantly a wino (which under our earlier assumption, $\theta_+^* = \theta_- = 0$, is $\tilde{\chi}_1$). The amplitude for this figure is

$$A = \frac{g^2}{M_c} d_L b_R \int \frac{d^4 k}{(2\pi)^2} \frac{K_{tb} K_{ud}^*}{[(k+p_b)^2 - \tilde{m}^2]^2} \frac{\Delta \tilde{m}_{L-R}^2}{[(k+p_d)^2 - \tilde{m}^2]} \frac{\bar{d}_L (k + m_{\tilde{\chi}_1}) b_L}{k^2 - m_{\tilde{\chi}_1}^2}, \quad (2.72)$$

where we have already set the squark masses equal to some common supersymmetric mass, but have left the chargino mass arbitrary. Using

$$\frac{1}{(k+p_b)^2 - \tilde{m}^2} \approx \frac{1}{k^2 - \tilde{m}^2} \left(1 - \frac{2k \cdot p_b}{k^2 - \tilde{m}^2} \right), \quad (2.73)$$

the amplitude can be simplified to

$$\begin{aligned} A &= \frac{g^2}{M_c} \bar{d}_L b_R \int \frac{d^4 k}{(2\pi)^4} \frac{K_{tb} K_{ud}^* \Delta \tilde{m}_{L-R}^2}{[k^2 - \tilde{m}^2]^4} \frac{(-4k \cdot p_b - 2k \cdot p_d) [\bar{d}_L \not{k} b_L]}{k^2 - m_{\tilde{\chi}_1}^2} \\ &= \frac{-g^2}{M_c} K_{tb} K_{ud}^* \Delta \tilde{m}_{L-R}^2 [\bar{d} R b] [\bar{d} (p_b + \frac{1}{2} p_d) L b] \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{[k^2 - \tilde{m}^2]^4 [k^2 - m_{\tilde{\chi}_1}^2]} \end{aligned} \quad (2.74)$$

Using $\Delta \tilde{m}_{L-R}^2 = m_t \tilde{m}$, $\bar{d} p_b L b = m_b \bar{d} R b$ and ignoring $\bar{d} p_d L b = m_d \bar{d} L b$, we have evaluating the integral

$$\mathcal{I}_{\text{eff}} = \frac{-1}{24\pi^2} \frac{G_F}{\sqrt{2}} \frac{m_t m_b M_W^2}{\tilde{m}^3 M_c} I_2(x) [\bar{d} R b] [\bar{d} R b], \quad (2.75)$$

where

$$I_2(x) = \frac{2 - 10x - 4x^2}{(1-x)^3} - \frac{12x^2 \ln x}{(1-x)^4} \rightarrow 1 \text{ as } x \rightarrow 1, \quad (2.76)$$

and $x = m_\chi^2 / \tilde{m}^2$.

For this effective Lagrangian we must solve for the matrix element using the Fierz identity $R \otimes R = -\frac{1}{2} R \otimes R - \frac{1}{8} R \sigma_{\mu\nu} \otimes \sigma^{\mu\nu}$, where the tensor term may be ignored. Thus we have that

$$\begin{aligned} \langle B | (\bar{d} R b) (\bar{d} R b) | \bar{B} \rangle &= B_B \left[\langle B | \bar{d} R b | 0 \rangle \langle 0 | \bar{d} R b | \bar{B} \rangle - \frac{1}{2} \cdot \frac{1}{3} \langle B | \bar{d} R b | 0 \rangle \langle 0 | \bar{d} R b | \bar{B} \rangle \right] \\ &= \frac{1}{4} \cdot \frac{5}{6} B_B \langle B | \bar{d} \gamma_5 b | 0 \rangle \langle 0 | \bar{d} \gamma_5 b | \bar{B} \rangle \\ &= \frac{1}{4} \cdot \frac{5}{6} \frac{f_B^2 m_B^2}{2 m_B} B_B (\rho_P^B)^2 = \frac{1}{4} \cdot \frac{5}{6} \frac{f_B^2 m_B^2}{2 m_B} B_B, \end{aligned} \quad (2.77)$$

where the last equality follows due to the fact that $\rho_P^B = 1$ (the ratio of constituent quark masses to current quark masses) for the B meson. The contribution of the supercomposite process to the mass difference is therefore,

$$\Delta m_{SC} = \frac{5}{576 \pi^2} \frac{m_t m_b M_W^2}{\tilde{m}^3 M_c} \frac{G_F}{\sqrt{2}} f_B^2 m_B B_B. \quad (2.78)$$

We are now in a position to calculate what effect supercomposite processes might have on the $B-\bar{B}$ system given the limit of $M_c = 2500$ TeV from K_L-K_S . What we calculate is $x_B = \tau^B \Delta m_B$, which from the Argus experiment [ARG.87] is $x_B = 0.73 \pm 0.18$. (Actually what they quote is a ratio of decay widths, $r = x_B^2 / (2 + x_B^2)$ [Hag.87a].) τ^B is the B meson lifetime. Setting $f_B = 1$ GeV, the QCD correction, $\eta_{QCD} = 0.84$, and $\tau^B = (0.598 \times 10^{-12} \text{ GeV})^{-1}$ we find for $x = m_\chi^2 / \tilde{m}^2 = 1$, $m_t = \tilde{m} = 100$ GeV that:

$$x_{SC} = \tau^B \Delta m_{SC} = \frac{5}{576 \pi^2} \eta_{QCD} \tau^B \frac{m_t m_b M_W^2}{\tilde{m}^3 M_c} \frac{G_F}{\sqrt{2}} f_B^2 m_B B_B = 0.001, \quad (2.79)$$

well below the experimentally observed value of 0.73. This value can be doubled if one considers $x = m_\chi^2 / \tilde{m}^2 = 0$ (i.e. a light ino mass). More importantly, the amplitude goes as the third power of the supersymmetric mass and therefore will fall rapidly for increased supersymmetric masses.

We should also calculate the dimension-six contribution,

$$x_C = \tau^B \Delta m_C = \frac{1}{4} \cdot \frac{5}{6} \eta_{QCD} \tau^B \frac{f_B^2 m_B B_B}{M_c^2} = 0.002. \quad (2.80)$$

This time the pure dimension-six term and the supercomposite effect are of the same

order. The supercomposite term will dominate for

$$M_c \geq \frac{24\pi^2}{G_F\sqrt{2}} \frac{\tilde{m}^3}{m_t m_b M_W^2} = 6500 \text{ TeV} \left(\frac{\tilde{m}}{100 \text{ GeV}} \right)^3. \quad (2.81)$$

In conclusion, we see that while one would a priori expect supercomposite effects scaled by a factor of $1/(\tilde{m}M_c)$ to dominate over pure composite effects which scale like $1/M_c^2$, the inclusion of two weak coupling constants and quark masses due to chiral constraints causes the supercomposite process to be suppressed. As well, given the stringent limits set on the composite scale by the K_L - K_S mass difference, it would appear that the B - $\bar{B}$ system does not improve on these limits and would be significant only if, for some reason, compositeness manifested itself only in the third generation of quarks. If this was the case, the K_L - K_S limit of 2,500 TeV might not apply to the composite mass scale in the B - $\bar{B}$ process, allowing the composite and supercomposite effects to be larger than those quoted above.

2.7 Concluding Remarks

In this chapter we found that it is possible in models with both supersymmetry and compositeness to have processes scaled by one power each of the composite and supersymmetric mass scales. If the supersymmetric mass scale is much lower than the composite mass scale, say on the order of the weak scale, then one should be able to set better constraints on the composite mass scale than could be set in a pure composite model where such effects are typically scaled by two powers of the composite mass scale.

From a decay chirally suppressed in the Standard Model, such as the decay of the pion into two leptons, one cannot constrain a supercomposite model significantly better than a composite model, the constraints on the composite mass scale from this process being 1,000 TeV (for a supersymmetric mass scale of 100 GeV) and 200 TeV respectively. In a non-flavour conserving situation as in muon conversion ($\mu N \rightarrow e N$), the

gains can be far better. Here one obtains a constraint on the composite mass scale of 60,000 TeV ($\tilde{m} \approx 100$ GeV), even the results for 1 TeV supersymmetric masses are almost an order of magnitude better than the 650 TeV constraint that can be imposed in a purely composite model.

Due to a need to exchange weak gauginos and a further problem with chiral constraints, the purely composite interactions that can lead to a K_L - K_S mass difference serve as a much more stringent test of compositeness than a supercomposite process. Here a purely composite process can constrain the composite mass scale to be greater than 2,500 TeV, while supercomposite process can only constrain it to 11 TeV. Supercomposite processes are more dominant in B - $\bar{B}$ mixing, however, current experimental constraints on this process do not significantly constrain either the supercomposite interactions or the purely composite interactions.

From the processes we have investigated to this point, the best constraint that can be put on purely composite models comes from the K_L - K_S mass difference. The best limit on supercompositeness comes from the lepton number violating process of muon conversion. Not surprisingly, both these constraints came from processes that violated Standard Model conservation laws. They might apply separately if one has composite leptons or quarks, but not both. One would have to resort to the π_{21} decay results if the compositeness did not break flavour-conservation laws, as might be the case if each generation of quarks and leptons was made of different constituents. At any rate all three processes apply to the first two generations of quarks and leptons. Only the B - $\bar{B}$ results tell us anything if compositeness manifested itself only in the third generation. In the case of π_{21} decay, the purely composite and the supercomposite constraints on the scale of compositeness are comparable for supersymmetric mass scales between 0.1 - 1 TeV. For muon conversion the constraints from supercompositeness are considerably better, while for the K_L - K_S mass difference the purely composite constraints are much better. A summary of the results can be found in figure 2.7 below.

Process	Pure Composite Limit on M_c	Supercomposite Limit on $\tilde{m}M_c$
π_{2I} Decay	200 TeV	$98 (\text{TeV})^2$
$\mu N \rightarrow e N$	650 TeV	$6000 (\text{TeV})^2$
$K_L - K_S$	2500 TeV	$1 (\text{TeV})^2$

Figure 2.7: A summary of the constraints put on both purely composite models and supercomposite models from the processes studied to date.

To this point we have investigated the effects of compositeness on several processes. The π_{2I} decay could test the effects of composite quarks that conserve flavour. Muon conversion, on the other hand, could test the effects of composite leptons that do not conserve flavour. Lastly, $K_L - K_S$ and $B - \bar{B}$ mixing could test the effects of composite quarks that do not conserve flavour, in particular, $B - \bar{B}$ mixing could test the effects of a composite third generation of quarks (and leptons). As of yet we have not investigated the possibility of composite objects that violate CP, nor have we investigated whether any of the gauge bosons, or their supersymmetric partners, might be composite objects. These things will be the topic of the next chapter.

3. COMPOSITE AND SUPERCOMPOSITE CONTRIBUTIONS TO THE NEUTRON ELECTRIC DIPOLE MOMENT

3.1 Introduction

So far we have concentrated on the effects of composite quarks and leptons. Furthermore, we have investigated flavour changing interactions, but not CP-violating interactions. In this chapter, we will investigate contributions to the neutron electric dipole moment from composite and supercomposite models. As such we will be investigating CP-violating interactions. However, more than that, we will be leaving the realm of two-fermion, two-scalar interactions to investigate possible form factors in gaugino interactions that are induced by compositeness, as well as possible new gaugino interactions. The former can be induced by composite quarks (or leptons), while the latter would require a composite gaugino. Thus this chapter will continue the investigation of the effects of composite quarks and leptons, but will also delve into the possible effects of composite gauginos. Due to the undesirability of composite massless gauge bosons [Cas.62, Wei.80], we will restrict the composite nature to gauginos. This may restrict the supersymmetric mass scale to be synonymous with the composite mass scale.

3.2 The Neutron Electric Dipole Moment

Of increasing interest of late have been constraints from the electric dipole moment of the neutron, d_n . This has been mainly due to Weinberg's introduction [Wei.89] of a previously unused three gluon CP-violating operator,

$$O_G \sim \frac{-1}{6} C f_{abc} G_{\mu\rho}^a G_\nu^{b\rho} G_{\lambda\sigma}^c \epsilon^{\mu\nu\lambda\sigma}. \quad (3.1)$$

It was initially thought [Dai.90a] that due to large QCD corrections, this operator would dominate over other CP-violating operators. However, more careful analysis of QCD corrections [Bra.90] suggested the quark colour and electric dipole operators might also

contribute. These operators are:

$$O_1^C = \frac{1}{2} C_1^C G_{\mu\nu}^a \bar{q} T_a \sigma^{\mu\nu} \gamma_5 q, \quad (3.2a)$$

$$O_2^C = \frac{1}{2} C_2^C (\bar{q}(x) \overleftrightarrow{\partial}^\mu \gamma_5 T^a q(x)) A_\mu^a(x) = -\frac{C_2^C}{2} \bar{q}(\bar{p}) (p + \bar{p})^\mu \gamma_5 T^a q(p) A_\mu^a, \quad (3.2b)$$

$$O^E = \frac{1}{2} C^E (\bar{q}(x) \overleftrightarrow{\partial}^\mu \gamma_5 q(x)) A_\mu(x) = -\frac{C^E}{2} \bar{q}(\bar{p}) (p + \bar{p})^\mu \gamma_5 q(p) A_\mu. \quad (3.2c)$$

O_2^C reduces to O_1^C on quark mass shell, so to the extent that one may consider the quarks in the neutron to be free, one can replace O_2^C by O_1^C , as well as replacing O^E by a similar operator. Using these operators, Weinberg and others were able to set constraints on the CP-violating portion of the Higgs sector [Wei.89, Boy.90], while others were able to set constraints on supersymmetric [Dai.90b, Arn.90, Arn.91] and left-right symmetric [Cha.90] theories. Finally, Dine and Fischler have calculated constraints on composite models and supersymmetry due to these operators [Din.90]

To actually calculate what effect each of these operators will have on the electric dipole moment of the neutron, we will use the naive dimensional analysis employed by Weinberg [Wei.89, Man.84, Geo.86]. This means

$$d_n^C = \frac{e}{4\pi} C_{1,2}^C, \quad (3.3a)$$

$$d_n^G = \frac{e}{4\pi} M_{\text{QCD}} C^G, \quad (3.3b)$$

and of course,

$$d_n^E = C^E, \quad (3.3c)$$

where each of the C's have been properly renormalized down to the hadronic scale. How each coefficient is renormalized will be discussed as the need arises.

3.3 Composite Contributions to the Neutron Electric Dipole Moment

What we first wish to study are possible contributions from compositeness to the neutron electric dipole moment, through one, or all, of the above operators. O^E , and the other dipole operators are dimension five and as a first guess one would expect $C^E \approx \frac{1}{M_C}$, where M_C is the composite mass scale. However, these operators flip chirality, and in so

much as we expect the compositeness to produce chiral quarks, we would expect

$$C^E = e \frac{m_q}{M_c^2}, \quad (3.4)$$

where m_q is a light quark mass. We can now find the contribution to d_n using a modified equation (3.3c), $d_n^E = \xi^E C^E$, where ξ^E is the renormalization group correction which runs the coefficient C^E from the top mass down to 1 GeV. Technically one should run the correction all the way down from the composite mass scale, but since the coupling constants run slowly above the top mass scale, any contributions from that area will yield no more than an $O(1)$ effect. Arnowitz, Lopez and Nanopoulos [Arn.90] have calculated ξ^E to be approximately 1.5. Assuming [Alt.86, Smi.90] $d_n \leq 10^{-25}$ e-cm $\approx e (1.44 \times 10^{12} \text{ GeV})^{-1}$, one has for $m_u = 8$ MeV that $M_c \geq 130$ TeV.

For the quark colour dipole moment, one would expect something similar to the electric dipole moment:

$$C^C = g_s \frac{m_q}{M_c^2}. \quad (3.5)$$

Using a modified equation (3.3a) that accounts for the QCD corrections, $d_n^C = \xi^C (e/4\pi) C^C$, with $\xi^C \approx 3.3$ [Arn.90], one finds for $g_s/4\pi \approx 0.1$, a constraint on the composite mass scale of $M_c \geq 60$ TeV from this process, a result not quite as restrictive as the more direct quark electric dipole term. More importantly, this colour dipole term for the heavier quarks can also give a contribution to the three gluon operator. One does not expect composite effects to lead to the three gluon operator, O_G , directly as the Case and Gasiorowicz [Cas.62], and Weinberg and Witten [Wei.80] theorem forbids composite massless gluons. However, the colour dipole term for heavy quarks leads to a three gluon term below the quark threshold. One has that the coefficient for the three gluon operator above and below a quark threshold is [Bra.90, Arn.90]

$$C^G(m_q) = C^G(m_q^*) + C_q^C(m_q) \frac{1}{8\pi} \frac{\alpha_s(m_q)}{m_q}. \quad (3.6)$$

Furthermore, each operator must be renormalized in going from one quark threshold to the

next,

$$C_q^G = \xi_q^G C_{q+1}^G, \quad (3.7a)$$

and

$$C_q^C = \xi_q^C C_{q+1}^C. \quad (3.7b)$$

The ξ_q are given by Arnowitz, Lopez and Nanopoulos [Arn.90]. The first set is $\xi_q^G = 0.8$ between the top and bottom, 0.92 between bottom and charm and 0.3 from the charm threshold to the hadronic scale, while the second set is $\xi_q^C = 1.29$ between the top and bottom, 1.14 between bottom and charm and 2.2 from the charm threshold to the hadronic scale. Also using $g_S(m_b) = 1.14 g_S(\tilde{m}_t)$ and $g_S(m_c) = 1.06 g_S(m_b)$, we have

$$C^G = 1.5 \frac{\alpha_S(m_t)}{8\pi} \frac{g_S(m_t)}{M_C^2}. \quad (3.8)$$

Using naive dimensional analysis [Man.84, Geo.86] to calculate the three gluon operator's contribution to the electric dipole moment, we have [Wei.89]

$$d_n \approx \frac{e}{4\pi} M_{QCD} C^G = \frac{1.5e}{2} \left(\frac{g_S(m_t)}{4\pi} \right)^3 \frac{M_{QCD}}{M_C^2}, \quad (3.9)$$

where M_{QCD} is the chiral symmetry breaking scale, $M_{QCD} = 2\pi f_\pi = 1,190$ MeV. This constrains the composite mass scale to be $M_C > 40$ TeV.

From purely composite effects, the most stringent constraint comes, not surprisingly, from a direct quark electric dipole term, which yields a constraint of 130 TeV. Can we do any better if one considers a supercomposite model?

3.4 Supercomposite Contributions to the Neutron Electric Dipole Moment

Once again, if we have a supersymmetric composite model there may exist operators involving supersymmetric particles that could be "dressed" to the operators O^G , O^C , and O^E . Furthermore, these operators may be scaled as $1/(\tilde{m}M_C)$ as opposed to $1/M_C^2$ for pure composite operators. Due to the nature of the operators leading to the neutron electric dipole moment, we can no longer dress two-scalar, two-fermion interactions to four-fermion interactions. We must consider other possible sources for the

supercompositeness to manifest itself.

We consider two places where supercompositeness might induce CP-violation. One is through a CP-violating form factor in the quark-squark-gluino coupling, for an effective Lagrangian

$$\mathcal{L}_{q\bar{q}\tilde{g}(\text{eff})} = -i\sqrt{2} g \phi_q^\dagger \bar{\lambda}_a (1 + ia\gamma_5) T_a q + \text{h.c.}, \quad (3.10)$$

where $a \approx \frac{m_{3/2}}{M_C} = \frac{\tilde{m}}{M_C}$ is the CP-violating form factor ($m_{3/2}$ is the mass of the gravitino).

The other CP-violating term we consider is a supersymmetric extension of the three gluon operator, a gluino colour dipole term [Arn.91],

$$\mathcal{L}_{\text{eff}} = \frac{i}{2} \frac{g_s}{M_C} f_{abc} \bar{\lambda}^a \sigma_{\mu\nu} G_b^{\mu\nu} \gamma_5 \lambda^c. \quad (3.11)$$

At this point we should make a brief comment on the supersymmetry breaking scale, which in turn determines the approximate masses of the supersymmetric particles that will be involved in the loop diagrams. Normally, supersymmetry is expected to solve the unnaturalness of the Higgs sector, and therefore we expect supersymmetric masses to be of the order of the weak scale. This is a good assumption for the supersymmetric masses if only the form factor vertices contribute. However, in the case of the gluino colour dipole operator, we have in essence chosen our gluinos to be composite objects. Since massless, composite gluons are forbidden [Cas.62, Wei.80], our composite gluino colour dipole operator cannot exist in a theory with unbroken supersymmetry. If supersymmetry is broken, as it must be, and this operator is induced at the composite scale, then one would expect the supersymmetric masses to be of the same order as the composite mass scale.

We are now ready to calculate the various contributions of our two CP-violating operators to the low energy operators O^G , O^C , and O^E . The effects of the CP-violating form factor are shown in figure 1. Notice that due to the chiral nature of O^C and O^E , we require a left-right chiral flip on the squark line. This flip, $\Delta\tilde{m}_{LR}^2 = m_q\tilde{m}$, is allowed due to mixing between the left and right squarks. Figure 1a yields an amplitude,

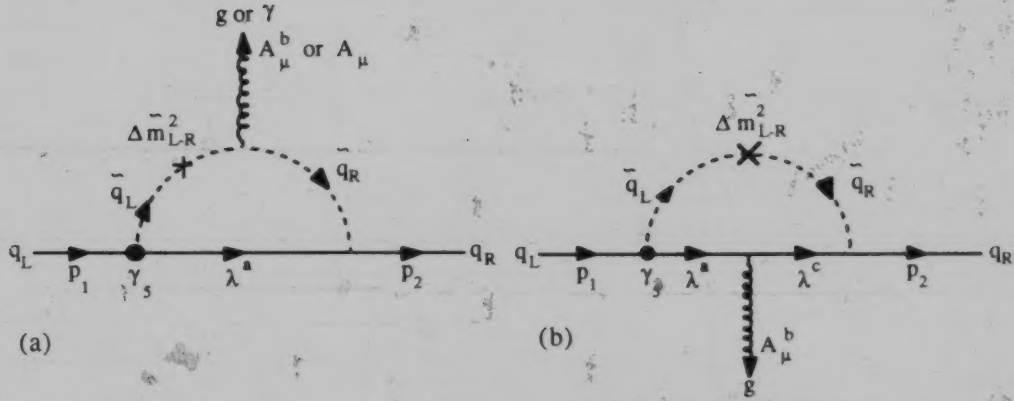


Figure 3.1: Two diagrams contributing to quark colour dipole through form factors at the quark-squark-gluino vertex. Figure 1a) can also contribute to the quark electric dipole by replacing the gluon with a photon. One also has similar diagrams with the left and right quarks interchanged.

$$A_1 \sim i 2 \cdot (\sqrt{2})^2 g_S^3 A_b^\mu \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{q}_R(p_2)(k+m_\lambda) a \gamma_5 T_a T_b T_a q_L(p_1)}{k^2 - m_\lambda^2} \Delta \tilde{m}_{L,R}^2 (-2k_\mu + (p_1 + p_2)_\mu) \times \left[\frac{1}{((k-p_1)^2 - \tilde{m}^2)^2} \frac{1}{((k-p_2)^2 - \tilde{m}^2)} + \frac{1}{((k-p_1)^2 - \tilde{m}^2)} \frac{1}{((k-p_2)^2 - \tilde{m}^2)^2} \right], \quad (3.12)$$

where the extra two comes from two possible positions to place the composite form-factor and the sum within the square brackets represents putting the chiral flip on either side of the gluon emission. Making use of the approximation

$$\frac{1}{(k+p_2)^2 - m^2} \frac{1}{(k+p_1)^2 - m^2} \approx \frac{1}{(k^2 - m^2)^2} \left(1 - \frac{2k \cdot (p_1 + p_2)}{k^2 - m^2} \right), \quad (3.13)$$

evaluating the integral, and adding left plus right contributions, one has

$$A_1 \sim -\frac{1}{18} g_S \left(\frac{g_S}{4\pi} \right)^2 \frac{\tilde{m}}{M_C} \frac{\Delta \tilde{m}_{L,R}^2}{\tilde{m}^3} \bar{q}(p_1 + p_2)_\mu \gamma_5 T_a q A_a^\mu, \quad (3.14)$$

where as usual, the integrals have been evaluated for a common supersymmetric mass, $m_\lambda = \tilde{m}$. We have also made use of the SU(3) identity (Appendix F), $T_a T_b T_a = -\frac{1}{6} T_b$.

Figure 1b yields an amplitude

$$A_2 \sim 2 \cdot 2 g_S^3 \int \frac{d^4 k}{(2\pi)^4} \bar{q}_R(p_2) \frac{(k+p_2+m_\lambda) \gamma_\mu (k+p_1+m_\lambda) a \gamma_5 T_a f_{abc} T_c}{((k+p_2)^2 - \tilde{m}^2)((k+p_1)^2 - \tilde{m}^2)} q_L(p_1) \frac{\Delta \tilde{m}_{L,R}^2 A_b^\mu}{(k^2 - \tilde{m}^2)^2}. \quad (3.15)$$

Simplifying this amplitude and adding left and right contributions, one has,

$$A_2 \sim -\frac{1}{2} g_s \left(\frac{g_s}{4\pi}\right)^2 \frac{\tilde{m}}{M_C} \frac{\Delta\tilde{m}_{L,R}^2}{\tilde{m}^3} \bar{q} (p_1+p_2)_\mu \gamma_5 T_a q A_a^\mu, \quad (3.16)$$

where this time the SU(3) identity used is $T_a f_{abc} T_c = -\frac{1}{2} i T_b$. Thus from the form factor contributions we arrive at, using $\Delta\tilde{m}_{L,R}^2 = m_q \tilde{m}$,

$$C_2^C = 2 \cdot \left(\frac{1}{18} + \frac{1}{2}\right) g_s \left(\frac{g_s}{4\pi}\right)^2 \frac{m_q}{\tilde{m} M_C} = \frac{10}{9} g_s \left(\frac{g_s}{4\pi}\right)^2 \frac{m_q}{\tilde{m} M_C}. \quad (3.17)$$

From figure 1a we can also deduce the electric dipole moment, replacing $T_a T_b T_a$ with $T_a T_a = \frac{4}{3}$, and a g_s with $\frac{2}{3}e$, we have

$$C^E = -\frac{16}{27} e \left(\frac{g_s}{4\pi}\right)^2 \frac{m_q}{\tilde{m} M_C}. \quad (3.18)$$

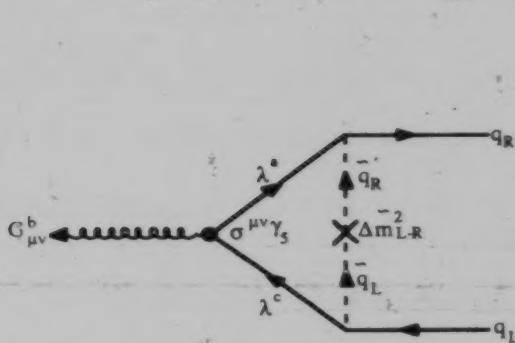


Figure 3.2: Diagram contributing to quark colour dipole through a gluino colour dipole term. Once again another diagram can be obtained by interchanging the left and right quarks.

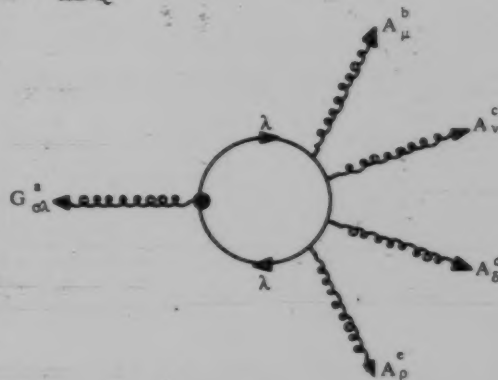


Figure 3.3: Diagram contributing to three gluon operator through a gluino colour dipole term.

There are also contributions to O_1^C from the gluino colour dipole operator. From figure 3.2, we have an amplitude

$$A \sim \frac{1}{2} \frac{2g_s^3}{M_C} \int \frac{d^4 k}{(2\pi)^4} \bar{q}_R \frac{1}{k-m_\lambda} \sigma_{\mu\nu} \gamma_5 \frac{1}{k-m_\lambda} f_{abc} T_a T_c q_L \frac{\Delta\tilde{m}_{L,R}^2}{(k^2 - \tilde{m}^2)^2} G_b^{\mu\nu}. \quad (3.19)$$

Adding left and right pieces and simplifying, one finds that

$$A \sim \frac{g_s}{4} \left(\frac{g_s}{4\pi}\right)^2 \frac{m_{u,d}}{\tilde{m} M_C} \bar{q} \sigma_{\mu\nu} \gamma_5 T_a q G_a^{\mu\nu}, \quad (3.20)$$

where once again, $\Delta\tilde{m}_{L,R}^2 = m_q \tilde{m}$ has been used. The gluino colour dipole contributes to the quark colour dipole a coefficient,

$$C_1^C = \frac{g_s}{2} \left(\frac{g_s}{4\pi} \right)^2 \frac{m_{u,d}}{\tilde{m}M_C} \quad (3.21)$$

Lastly, the gluino colour dipole operator also contributes to the three gluon operator. Figure 3.3 contributes an amplitude

$$A \sim \frac{1}{2} \frac{g_s^5}{M_C} G_{\sigma\lambda}^a A_\mu^b A_\nu^c A_\rho^d A_\rho^e f_{fab} f_{hbf} f_{ich} f_{jdi} f_{gej} \quad (3.22)$$

$$\int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[\sigma^{\sigma\lambda} \gamma_5 \frac{1}{\not{k}-m_\lambda} \gamma^\mu \frac{1}{\not{k}-m_\lambda} \gamma^\nu \frac{1}{\not{k}-m_\lambda} \gamma^\delta \frac{1}{\not{k}-m_\lambda} \gamma^\rho \frac{1}{\not{k}-m_\lambda} \right],$$

which after a lengthy calculation yields (Appendix G)

$$A \sim \frac{6i}{16\pi^2} \frac{1}{\tilde{m}M_C} g_s^3 f_{abc} G_{\mu\rho}^a (f_{bde} A_\nu^d A_\rho^e) (f_{cfg} A_\lambda^f A_\rho^g) \epsilon^{\mu\nu\lambda\sigma} \quad (3.23)$$

in the final analysis. It thus produces the six gluon (momentum independent) portion of the three gluon operator

$$O^G \sim \frac{1}{6} C^G f_{abc} G_{\mu\rho}^a G_\nu^b G_{\lambda\sigma}^c \epsilon^{\mu\nu\lambda\sigma}, \quad (3.24)$$

with

$$C^G = 6g_s \left(\frac{g_s}{4\pi} \right)^2 \frac{1}{\tilde{m}M_C} \quad (3.25)$$

We are now ready to find the constraints on the composite mass scale that d_n imposes on these operators. However, care must be taken to renormalize the operators down from the scale they first appear at - namely the supersymmetric scale, to the low energy hadronic scale. Unfortunately, as was mentioned earlier, we do not really know at what scale the supersymmetric masses first appear. However, for renormalization purposes we will assume all our operators become effective operators around 100 GeV, not so much because that is where we believe that they become effective operators, but rather because the slow running of the strong coupling above the top mass leads to a small ($\sim O(1)$) renormalization of the operators beyond this scale.

Moving on to the calculated coefficients. To renormalize C^E and C^C from the top scale to the hadronic scale, one has

$$C^i(m_t) = \xi^i C^i(\mu), \quad (3.26)$$

where $\xi^E = 1.5$ and $\xi^C = 3.3$ have already been quoted [Arm.90]. Now renormalized, naive dimensional analysis can be used to estimate the operator's contribution to d_n ,

$$d_n^E = \xi^E \frac{16}{27} e \left(\frac{g_s(\tilde{m})}{4\pi} \right)^2 \frac{m_u}{\tilde{m}M_C}. \quad (3.27)$$

Using [Alt.86, Smi.90] $d_n \leq 10^{-25} e \cdot \text{cm} = e (1.44 \times 10^{12} \text{ GeV})^{-1}$ to set limits on the operator, one obtains a constraint on the form factors. Using $\frac{g_s(\tilde{m})}{4\pi} = \frac{g_s(m_u)}{4\pi} = 0.1$ and $m_u \approx 8 \text{ MeV}$, we find that $\tilde{m}M_C \geq 1 \times 10^8 (\text{GeV})^2$. For the renormalized colour dipole term, one has a contributions to d_n of

$$d_n^C = \frac{e}{4\pi} \xi^C C_{1,2}^C(\tilde{m}) = e \xi^C \frac{10}{9} \left(\frac{g_s(\tilde{m})}{4\pi} \right)^3 \frac{m_u}{\tilde{m}M_C}. \quad (3.28)$$

which yields a lesser constraint of $\tilde{m}M_C \geq 4 \times 10^7 (\text{GeV})^2$.

The gluino colour dipole operator only contributes to C_1^C and C^G . From the quark colour dipole term we have

$$d_n^C = \frac{e}{4\pi} \xi^C C_1^C = \frac{e \xi^C}{2} \left(\frac{g_s(\tilde{m})}{4\pi} \right)^3 \frac{m_u}{\tilde{m}M_C}, \quad (3.29)$$

imposing a constraint $\tilde{m}M_C \geq 1.9 \times 10^7 (\text{GeV})^2$. Finally, we can calculate the colour dipole term's contribution to d_n through the three gluon operator. To calculate this contribution we must also include contributions from the quark colour dipoles at each quark threshold we pass. One has

$$C^G(m_q) = C^G(m_q^+) + C_q^C(m_q) \frac{1}{8\pi} \frac{\alpha_s(m_q)}{m_q}, \quad (3.30)$$

however, since C^C and C^G are both proportional to $\left(\frac{g_s}{4\pi}\right)^3$ the contributions to C^G from the quark threshold are suppressed by $\left(\frac{g_s}{4\pi}\right)^2$, making contributions from the heavy quarks inconsequential. One must still renormalize the operator C^G from the supersymmetric scale to the hadronic scale. Using the same renormalization scheme we used earlier, we have

$$C^G(\mu) = 0.2 C^G(m_t) = 0.2 C^G(\tilde{m}), \quad (3.31)$$

where μ is the hadronic scale. Finally,

$$d_n^G = \frac{e}{4\pi} M_{\text{QCD}} C^G(\mu) = e (0.2) \cdot 6 M_{\text{QCD}} \left(\frac{g_s(m_t)}{4\pi} \right)^3 \frac{1}{\tilde{m}M_C}, \quad (3.32)$$

which constrains the product of the supersymmetric and composite mass scales to be $\tilde{m}M_C \geq 2 \times 10^9 (\text{GeV})^2$.

We now have two constraints, $\tilde{m}M_C \geq (1 \times 10^8 \text{ and } 2 \times 10^9) (\text{GeV})^2$ from a CP-violating form factor and a gluino colour dipole term. What constraints can be set on the composite mass scale from these? If we take the supersymmetric mass scale to be of order 100 GeV, one has $M_C \geq (1,000 \text{ and } 20,000) \text{ TeV}$, respectively. However, only the first of these should be taken seriously, as the gluino colour dipole term requires a composite gluino. If, on the other hand, the supersymmetric mass is of order the composite mass, which may well be the case if gluinos are composite, we have $M_C \geq (10 \text{ and } 50) \text{ TeV}$ respectively.

3.5 Concluding Remarks

In conclusion, once again we have set limits on the scale of compositeness in a pure composite theory, and one with supersymmetry. From the CP-violating neutron electric dipole moment, d_n , we have found limits on the scale of supercompositeness ranging from 1,000 to 20,000 TeV if the supersymmetric scale is of order the weak scale and from 10 to 50 TeV if the supersymmetric scale is of order the composite scale. These limits are at least as restrictive and possibly quite a bit more restrictive than the limit of 130 TeV set from pure composite contributions to the electric dipole moment of the neutron. The limit of 1,000 TeV on the scale of compositeness (for a supersymmetric scale of order 100 GeV) would apply to composite quarks with a CP-violating form factor contribution to gaugino couplings. The 50 TeV limit would apply to a theory with composite gluinos (but not composite gluons) where the composite and supersymmetric scales are of the same order.

We may now wish to compare these results to the results of the previous chapter. In this chapter we obtained constraints on purely composite models from the quark electric dipole operator and the 3 gluon operator. Both of these were a test on the compositeness of quarks, as the 3-gluon operator was derived from heavy quark colour dipole operators. As such they are closely akin to the results from the K_L - K_S mass difference, except this time

the compositeness involves CP-violating effects as opposed to flavour changing effects. As one can see, the results here are an order of magnitude down from the K_L - K_S results, however, that may be due to the inclusion of strong coupling constants in the effective Lagrangian. The constraint on supercompositeness from the quark electric dipole moment is also a constraint involving compositeness, or rather supercompositeness, in quarks, as that was derived from the CP-violating form factor in the quark-squark-gluino coupling. Again this is quite similar in nature to our results from the K_L - K_S mass difference, but this time the constraint on the supercomposite mass scale of 1,000 TeV, for a supersymmetric mass scale of 100 GeV, does a much better job of probing supercompositeness in quarks.

The results from the gluino colour dipole are considerably different from anything we have done in the past. It is essentially a probe of compositeness in the gauginos, but not the gauge bosons. The fact that we differentiate between the gauginos and gauge bosons leads to our requirement that the supersymmetric and composite mass scales must be synonymous. As a result, although the constraint of 50 TeV obtained on supercompositeness is rather weak, it is a probe of a particular supercomposite nature that has not as yet received much attention. A summary of the results from chapters 2 and 3 can be found in figure 3.4.

Process	Pure Composite Limit on M_c	Supercomposite Limit on $\tilde{m}M_c$
π_{2I} Decay	200 TeV	$98 (\text{TeV})^2$
$\mu N \rightarrow e N$	650 TeV	$6,000 (\text{TeV})^2$
$K_L - K_S$	2,500 TeV	$1 (\text{TeV})^2$
NEDM (quark electric dipole)	130 TeV	$100 (\text{TeV})^2$
NE (3 gluon operator)	40 TeV (from quark colour dipole)	$2,000 (\text{TeV})^2$

Figure 3.4 A summary of the constraints on purely composite and supercomposite models studied in chapters 2 and 3.

This will conclude our analysis of supercomposite processes. We have now covered most of the possible scenarios involving composite natures for particles now assumed to be elementary. We have found that for the most part, in theories with both supersymmetry and a composite nature, one can set much more stringent constraints on the scale of compositeness than one could set in purely composite theories.

4. EFFECTS OF ANOMALOUS $WW\gamma$ COUPLINGS ON $b \rightarrow s\gamma$ AND $b \rightarrow s\mu^+\mu^-$

4.1 Introduction

So far this thesis has dealt mainly with supercomposite theories. We are now going to do something different, as this chapter will be devoted to the study of how anomalous contributions to the $WW\gamma$ vertex affect the decays $b \rightarrow s\gamma$ and $b \rightarrow s\mu^+\mu^-$. In particular, we will investigate the effect of a anomalous quadrupole coupling. Such an coupling could be induced by composite models of the massive gauge bosons, as we shall see in the next section. However, we shall investigate this coupling in a model independent fashion.

By imposing C and P symmetries separately and ignoring the scalar component of the vector boson ($\partial_\mu V^\mu$ and $\partial_\mu W^\mu$), one can write down the most general Lagrange density involving two charged and one neutral vector bosons. It is [Kan.89]:

$$\mathcal{L}_{WWV} = ie g_{WWV} [(W_{\mu\nu}^\dagger W^\mu V^\nu - W_\mu^\dagger V_\nu W^{\mu\nu}) + \kappa_V W_\mu^\dagger W_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W_{\lambda\mu}^\dagger W^\mu V^{\nu\lambda}], \quad (4.1)$$

where $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. W^μ is the W field, and V is either γ or Z , with $g_{WW\gamma} = 1$ and $g_{WWZ} = \cot \theta_W$. The Standard Model values are $\kappa_\gamma = \kappa_Z = 1$, and $\lambda_\gamma = \lambda_Z = 0$. One can see that in addition to a non-Standard Model anomalous magnetic moment parameter, κ_γ , one can also have a dimension six anomalous quadrupole coupling, λ_γ . It is the effects of a possible quadrupole coupling that we wish to investigate.

4.2 Anomalous $WW\gamma$ Couplings from Composite Gauge Bosons.

Instead of starting with equation (1.29) and adding to it a current mixing term (equation (1.31)), one can summarize the global $SU(2)$ invariance approach by the Lagrangian

$$\mathcal{L} = \frac{1}{4} \tilde{W}_{\mu\nu} \cdot \tilde{W}^{\mu\nu} + g \tilde{W}_\mu \cdot j^\mu + \frac{1}{2} M_W^2 \tilde{W}_\mu \cdot \tilde{W}^\mu - \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} (1 - \lambda^2) + e \tilde{A}_\mu j^\mu, \quad (4.2)$$

with the substitution rule

$$\tilde{W}_\mu^1 \equiv W_\mu^1, \quad \tilde{W}_\mu^2 \equiv W_\mu^2, \quad \tilde{W}_\mu^3 = W_\mu^3 + \lambda \tilde{A}_\mu. \quad (4.3)$$

Proceeding with the diagonalization (equation (1.32)), one once again obtains the Standard Model neutral current Lagrangian (equation (1.34)).

Since one is dealing with a composite theory, one need not just introduce renormalizable terms. To the above Lagrangian one can consider adding a trilinear vector boson interaction containing not only dimension-four terms, but dimension-six terms as well. As with the original Lagrangian it will be globally SU(2) invariant in the absence of electromagnetism [Kur.87]:

$$\begin{aligned} \mathcal{L}_{\text{tril}} &= -\frac{1}{2} \hat{g} W_{\mu\nu} \cdot (W^\mu \times W^\nu) - \frac{\rho_W}{6M_W^2} W^{\nu\lambda} \cdot (W_{\lambda\mu} \times W^\mu{}_\nu) \\ &= i\hat{g} W_\mu^3 (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) + i\hat{g} W_{\mu\nu}^3 W^{+\mu} W^{-\nu} + \frac{i\rho_W}{M_W^2} W^{3\nu\lambda} W_{\lambda\mu}^+ W^{-\mu}{}_\nu. \end{aligned} \quad (4.4)$$

Applying the substitution rule (4.3), $W_\mu^3 \rightarrow W_\mu^3 + \lambda \tilde{A}_\mu$, the electromagnetic interaction implied by this trilinear interaction is

$$\mathcal{L}_{\text{em}} = i\lambda \hat{g} \tilde{A}_\mu (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) + i\hat{g}\lambda \tilde{F}_{\mu\nu} W^{+\mu} W^{-\nu} + \frac{i\lambda \cdot \rho_W}{M_W^2} \tilde{F}^{\nu\lambda} W_{\lambda\mu}^+ W^{-\mu}{}_\nu. \quad (4.5)$$

Requiring the correct coupling of the photon to charge ± 1 W's after diagonalization according to equation (1.32) imposes that

$$\lambda \hat{g} = \left(\frac{e}{g}\right) \hat{g} = e, \quad (4.6)$$

or $\hat{g} = g$. This then implies that the anomalous magnetic moment is $\kappa_\gamma = 1$.

There is a problem with the above trilinear gauge boson interaction, equation (4.4). The problem is that the electromagnetic interaction suggested by equation (4.5) is not electromagnetically gauge invariant as it must be under the transformations

$$\tilde{A}_\mu \rightarrow \tilde{A}_\mu + \partial_\mu \alpha(x), \quad W_\mu^\pm \rightarrow \exp[\pm i g \alpha(x)] W_\mu^\pm. \quad (4.7)$$

To achieve electromagnetic invariance, one needs to start with a Lagrangian that apart from being globally SU(2) invariant, also has a local U(1) invariance around the third axis in weak isospin space:

$$W_\mu^3 \rightarrow W_\mu^3 + \lambda \partial_\mu \alpha(x) \equiv W_\mu^3 + \partial_\mu \omega(x), \quad W_\mu^\pm \rightarrow \exp[\pm i g \omega(x)] W_\mu^\pm. \quad (4.8)$$

Kuroda, Renard and Schildknecht assert that global SU(2) invariance along with a local U(1) invariance around a preferred axis implies local SU(2) invariance [Kur.87]. The trilinear gauge boson interaction must be appropriately generalized to

$$W_{\mu\nu} \rightarrow W_{\mu\nu} + g W_\mu \times W_\nu. \quad (4.9)$$

Applying this to the kinetic energy and quadrupole terms, one has the W self-interaction Lagrangian

$$\begin{aligned} \mathcal{L} = & M_W^2 W^\mu \cdot W_\mu - \frac{1}{4} (W_{\mu\nu} + g W_\mu \times W_\nu)^2 \\ & - \frac{\rho_W}{6M_W^2} (W^{\nu\lambda} + g W^\nu \times W^\lambda) \cdot [(W_{\lambda\mu} + g W_\lambda \times W_\mu) \times (W^\mu_\nu + g W^\mu \times W_\nu)]. \end{aligned} \quad (4.10)$$

Considering only the trilinear terms, one notices the only difference between (4.4) and (4.10) is that one has explicitly imposed $\hat{g} = g$ to accompany the gauge invariance. The trilinear electromagnetic interaction is therefore just equation (4.5) with $\hat{g} = g$. Diagonalizing (4.4) and (4.5) using the prescription of equation (1.32), one has the trilinear vector boson interaction

$$\begin{aligned} \mathcal{L}_{WW(\gamma,Z)} = & ie \left\{ [A_\mu (W^{\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu) + A_{\mu\nu} W^{+\mu} W^{-\nu} + \frac{\rho_W}{gM_W^2} \tilde{F}^{\nu\lambda} W_{\lambda\mu}^+ W^{-\mu}_\nu] \right. \\ & \left. + \cot \theta_W [Z_\mu (W^{\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu) + Z_{\mu\nu} W^{+\mu} W^{-\nu} + \frac{\rho_W}{gM_W^2} Z^{\nu\lambda} W_{\lambda\mu}^+ W^{-\mu}_\nu] \right\}. \end{aligned} \quad (4.11)$$

Comparing this with equation (4.1), one sees that $\kappa_\gamma = 1$, as we have already mentioned, and $\lambda_\gamma = \lambda_Z = \rho_W/g$, with ρ_W being an arbitrary parameter. Therefore models of this type impose $\kappa_\gamma = 1$, but leave $\lambda_\gamma = \lambda_Z$ arbitrary. It is just such a case that we wish to investigate.

4.3 Constraints on the Anomalous W Quadrupole Coupling

The study of the WW γ coupling, as parametrized in equation (4.1), has been a subject of intense interest lately,¹ mainly due to the fact that the experimental facilities are just now coming on line to test it. The two photon couplings; the anomalous magnetic

¹ See for example references [Bau.88a,b, Bau.90, Bel.90, Cou.91, God.91, Gra.85, Grif.88, Hag.87b, Kan.89, Wal.85].

moment parameter, κ_Z , and the anomalous quadrupole coupling, λ_γ , contribute to the magnetic dipole moment of the W boson, μ_W ,

$$\mu_W = \frac{e}{2M_W} (1 + \kappa_\gamma + \lambda_\gamma), \quad (4.12)$$

and the electric quadrupole moment, Q_W ,

$$Q_W = -\frac{e}{M_W^2} (\kappa_\gamma - \lambda_\gamma). \quad (4.13)$$

In this thesis we shall only be concerned with the dimension-six anomalous quadrupole coupling, λ , and in particular, λ_γ .

The most restrictive constraint on λ comes from the mass shifts of gauge bosons (the ρ parameter). Corrections to the ρ parameter from these couplings are proportional to $(\lambda_\gamma - \lambda_Z) \Lambda^4$ [Kan.89, Grif.88], where Λ is the scale of new physics (compositeness). The Case and Gasiorowicz, Weinberg and Witten theorem [Cas.62, Wei.80] imposes that $M_W \geq g\Lambda$ [Pes.81], and therefore, Λ cannot be much larger than 1 TeV. For Λ of order 1 TeV, the above correction to the ρ parameter imposes that $\lambda_\gamma = \lambda_Z$.

Constraints on the magnitude of each coupling are not as well constrained. One of the few results independent of the cutoff scale, Λ , comes from g-2 of the muon [Gra.85, Wal.85]. It imposes that $-2 \leq \lambda_\gamma \leq 6$. Unitarity constraints on the anomalous vertex from $f\bar{f} \rightarrow WW, WZ$ and $W\gamma$ [Bau.88a] impose $|\lambda| \leq 0.6 \left(\frac{1\text{TeV}}{\Lambda}\right)^2$. Obviously this last result is quite scale dependent, so the scale independent result from g-2 of the muon remains one of the stronger constraints. Logarithmically scale dependent results may be obtained through direct searches. From Tevatron limits on $p\bar{p} \rightarrow W\gamma$, one can expect to obtain $|\lambda| < 0.25 - 0.3$ [Bau.90] and from the ratio of the hadronic to muon decay widths at LEP, one can achieve $|\lambda| \leq 0.34$ [Bel.90]. Results from future colliders will be better, $|\lambda| \leq 0.054$ and 0.039 for the LHC and SSC respectively [Bau.88b]. A possible 500 GeV e^+e^- collider could measure $|\lambda| \leq 0.07$ [Cou.91].

Some of these scale dependent constraints are quite good for all reasonable values of the scale of new physics, Λ . Indeed, one might argue that the constraints that are

logarithmically dependent on this scale are, for all intensive purposes, scale independent. Still, further scale independent results are clearly needed. Flavour changing interactions, such as $b \rightarrow s$ transitions will lead to scale independent results and therefore would be a good place to investigate the effects of an anomalous $WW\gamma$ coupling.

These flavour-changing decays of the b quark to an s quark have been a subject of much interest of late. These decays, prohibited at tree level in the Standard Model, not only test the subtleties of the Standard Model, but also serve as a good test of more exotic physics. Furthermore, since the decay proceeds through a top quark loop, it should provide some constraints on the top quark mass. Finally, with the coming age of B factories, the Standard Model predictions for $b \rightarrow s$ decays should be put to the test. Already the CLEO [Ave.89] and ARGUS [ARG.89] collaborations have provided upper bounds to these decays, proving the Standard Model cannot be too far wrong.

Standard Model calculations of the decays of $b \rightarrow s\gamma$ ² and $b \rightarrow s\mu^+\mu^-$ [Grig.89, Grin.89] are in the literature, along with calculations of non-Standard Model decays for supersymmetric theories [Ber.87b], extended Higgs models [Ell.86, Hou.88, Riz.88, Grin.88b, Grin.89], and for four generations [Hew.87, Hou.87]. Of particular interest to the calculation that will be performed here, S.P. Chia [Chi.90] has studied effects due to the $WW\gamma$ coupling, and specifically effects due to a non-Standard Model, W magnetic moment parameter, $\kappa_\gamma \neq 1$.

4.4 Anomalous Contributions to $b \rightarrow s\gamma$

Putting κ to its Standard Model value of 1 in the Lagrange density of equation (4.1), one can write down the addition to the $WW\gamma$ vertex function with incoming or outgoing fields $A_\sigma(p)$, $W_\alpha(q)$ and $W_\beta^+(\bar{q})$ (figure 4.1) from the anomalous quadrupole terms, λ_γ :

²See references [Cam.82, ODo.86, Ber.87a, Des.87, Alt.88, Des.88, Grig.88, Grin.88a].

$$\Gamma_{\sigma\alpha\beta}(p,q,\bar{q}) = ie \frac{\lambda_\gamma}{M_W^2} \{ q \cdot \bar{q} (p_\beta g_{\alpha\sigma} - p_\alpha g_{\beta\sigma}) + \bar{q}_\alpha (q \cdot p g_{\beta\sigma} - q_\sigma p_\beta) + q_\beta (\bar{q}_\sigma p_\alpha - \bar{q} \cdot p g_{\alpha\sigma}) + g_{\alpha\beta} (\bar{q} \cdot p q_\sigma - \bar{q}_\sigma q \cdot p) \}. \quad (4.14)$$

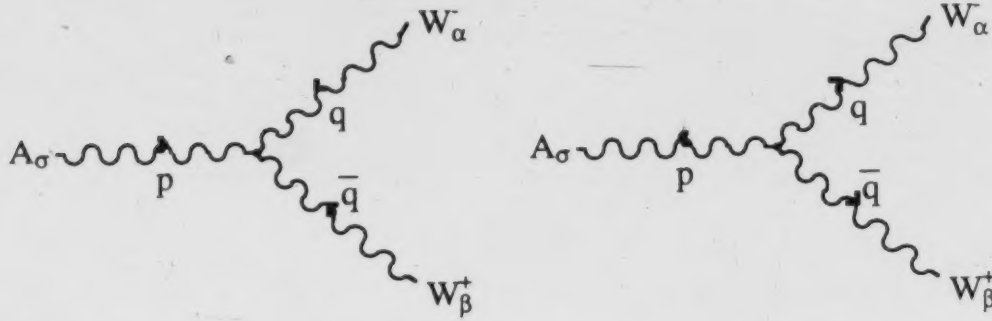


Figure 4.1: The $WW\gamma$ vertex for incoming or outgoing fields. The photon or Z carries momentum p , while the W^- (W^+) carries momentum q ($\bar{q}$). The vertex function is the same for incoming or outgoing fields since $\Gamma_{\sigma\alpha\beta}(-p,-\bar{q},-q) = -\Gamma_{\sigma\alpha\beta}(p,\bar{q},q) = \Gamma_{\sigma\alpha\beta}(p,q,\bar{q})$.

Observe that $q_\alpha \Gamma_{\sigma\alpha\beta} = \bar{q}_\beta \Gamma_{\sigma\alpha\beta} = p_\sigma \Gamma_{\sigma\alpha\beta} = 0$, this will ensure that the longitudinal portion of the W propagators, and thus the gauge dependent portions, do not contribute. We are interested in the transition $b \rightarrow s\gamma^*$ which proceeds through a $WW\gamma$ vertex. This is illustrated in figure 4.2 and the corresponding vertex function, ignoring the longitudinal portions of the W propagators, which do not contribute, is:

$$\Gamma_\mu = i \frac{g^2}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{s}(p_2) \gamma^\alpha k \gamma^\beta \left(\frac{1-\gamma_5}{2} \right) (K_{js}^* K_{jb}) b(p_1)}{[k^2 - m_f^2][(k-p_1)^2 - M_W^2][(p_2-k)^2 - M_W^2]} \Gamma_{\mu\alpha\beta}(q, (p_2-k), (k-p_1)). \quad (4.15)$$

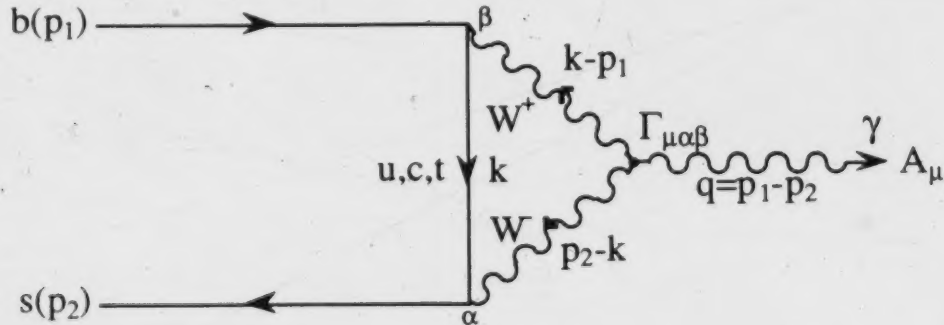


Figure 4.2: Feynman diagram for $b \rightarrow s\gamma^*$ with a $WW\gamma$ vertex.

One now performs the standard Feynman parametrization:

$$\Gamma_\mu = i g^2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{(2\pi)^4} \bar{s}(p_2) \gamma^\alpha k \gamma^\beta \left(\frac{1-\gamma_5}{2} \right) (K_{js}^* K_{jb}) b(p_1) \Gamma_{\mu\alpha\beta}(q, p_2-k, k-p_1) \quad (4.16)$$

$$\frac{1}{[k^2 - 2k \cdot (p_1 y + p_2(1-x-y)) + p_1^2 y + p_2^2(1-x-y) - m_j^2 x - M_W^2(1-x)]^3}$$

and makes the substitution $k' = k - [p_1 y + p_2(1-x-y)]$, so that one gets

$$\Gamma_\mu = i g^2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k'}{(2\pi)^4} \frac{\bar{s}(p_2) \gamma^\alpha [k' + p_1 y + p_2(1-x-y)] \gamma^\beta \left(\frac{1-\gamma_5}{2} \right) (K_{js}^* K_{jb}) b(p_1)}{[k'^2 - M^2(x, y)]^3} \Gamma_{\mu\alpha\beta}(q, -k' - qy + p_2 x, k' - q(1-y) - p_2 x), \quad (4.17)$$

where

$$M^2(x, y) = [p_1 y + p_2(1-x-y)]^2 - p_1^2 y - p_2^2(1-x-y) + m_j^2 x + M_W^2(1-x). \quad (4.18)$$

Keeping only terms quadratic in k' , since cubic terms do not survive the k' integration and lower order terms will be small, one finds after significant simplification (Appendix H),

$$\Gamma_\mu = \frac{eg^2 \lambda_\gamma}{2M_W^2} \int_0^1 dx \int_0^{1-x} dy \bar{s}(p_2) \{ (3x-1) i \sigma_{\mu\nu} q^\nu (m_b R + m_s L) + (3-3x)(q_\mu \not{q} - q^2 \gamma_\mu) L \} (K_{js}^* K_{jb}) b(p_1) \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{[k^2 - M^2(x, y)]^3} \quad (4.19)$$

R and L are the usual right and left projection operators, $\frac{1}{2}(1 \pm \gamma_5)$, respectively. Performing the k integration (Appendix D), one has

$$\Gamma_\mu = \frac{-ieg^2 \lambda_\gamma}{32\pi^2 M_W^2} \int_0^1 dx \int_0^{1-x} dy \bar{s}(p_2) \{ (3x-1) i \sigma_{\mu\nu} q^\nu (m_b R + m_s L) + (3-3x)(q_\mu \not{q} - q^2 \gamma_\mu) L \} K_{js}^* K_{jb} b(p_1) \cdot \ln \frac{M^2(x, y)}{\Lambda^2}, \quad (4.20)$$

where Λ is the cutoff for the divergent k integration and terms with no $M(x, y)$, and therefore no m_j , dependence have not been included, as they would disappear when the amplitude is summed over the internal quarks (GIM cancellation). The scale dependence will also disappear upon summation over internal quarks. To evaluate the x and y integrals, it is convenient to assume $p_1^2, p_2^2, q^2 \ll M_W^2$ so that $M^2(x, y)$ becomes,

$$M^2(x, y) = M^2(x) = M_W^2 \left[1 + x \left(\frac{m_j^2}{M_W^2} - 1 \right) \right]. \quad (4.21)$$

The integral is now easy to evaluate, and the final result summing over internal quarks is

$$\Gamma_\mu = \frac{-ie G_F}{4\pi^2 \sqrt{2}} \sum_j K_{js}^* K_{jb} \{ A_\lambda(z_j)(\bar{q}_\mu \not{q} - q^2 \gamma_\mu)L + iB_\lambda(z_j)\sigma_{\mu\nu}q^\nu(m_s L + m_b R) \}, \quad (4.22)$$

where $z_j = m_j^2/M_W^2$,

$$A_\lambda(z_j) = \frac{-z_j(3z_j-1)}{2(z_j-1)^2} + \frac{z_j^2 \ln(z_j)}{(z_j-1)^3}; \quad A_\lambda(1) = \frac{1}{3}, \quad (4.23)$$

and

$$B_\lambda(z_j) = \frac{z_j(z_j+1)}{2(z_j-1)^2} - \frac{z_j^2 \ln(z_j)}{(z_j-1)^3}; \quad B_\lambda(1) = \frac{1}{6}. \quad (4.24)$$

In the calculations of A_λ and B_λ , constants have been added so that $A_\lambda(0) = B_\lambda(0) = 0$.

Summing these constants over internal quarks gives zero according to the GIM mechanism.

We are now in a position to see what effect this anomalous coupling will have on $b \rightarrow s\gamma$. A general vertex function for $b \rightarrow s\gamma^*$ can be written as

$$\Gamma_\mu(q) = \frac{ie G_F}{4\sqrt{2}\pi^2} \sum_j K_{js}^* K_{jb} [A(z_j)(\bar{q}_\mu \not{q} - q^2 \gamma_\mu)L + iB(z_j)\sigma_{\mu\nu}q^\nu(m_s L + m_b R)]. \quad (4.25)$$

The Standard Model values for A and B have been calculated. For B, one has [Cam.82, Chi.90, Ina.81]:

$$B_{SM} = \frac{z_j(8z_j^2+5z_j-7)}{12(z_j-1)^3} - \frac{z_j^2(3z_j-2)}{2(z_j-1)^4} \ln(z_j); \quad B_{SM}(1) = \frac{5}{24}. \quad (4.26)$$

A is a gauge dependent quantity, and has the Standard Model values

$$A_{SM} = \frac{z_j(82-151z_j+63)}{36(z_j-1)^3} + \frac{6z_j^4-123z_j^3+234z_j^2-127z_j+16}{36(z_j-1)^4} \ln z_j; \quad A_{SM}(1) = \frac{85}{108}, \quad (4.27)$$

in the unitary gauge [Chi.90], and

$$A_{SM} = \frac{z_j^2(19z_j-25)}{36(z_j-1)^3} + \frac{6z_j^4-60z_j^3+108z_j^2-64z_j+16}{36(z_j-1)^4} \ln z_j; \quad A_{SM}(1) = \frac{-11}{216}, \quad (4.28)$$

in the Feynman gauge [Grig.89, Grin.89]. The gauge dependent portion can be found in

[Ina.81], however, their value for A_{SM} seems to differ from the other two. The results

from the anomalous couplings can be added simply by taking $A_{tot} = A_{SM} - \lambda A_\lambda$ and

$B_{tot} = B_{SM} - \lambda B_\lambda$. Figure 4.3a gives B_λ and B_{SM} as functions of top mass, while figure

4.3b does the same for A_λ and A_{SM} in the two different gauges.

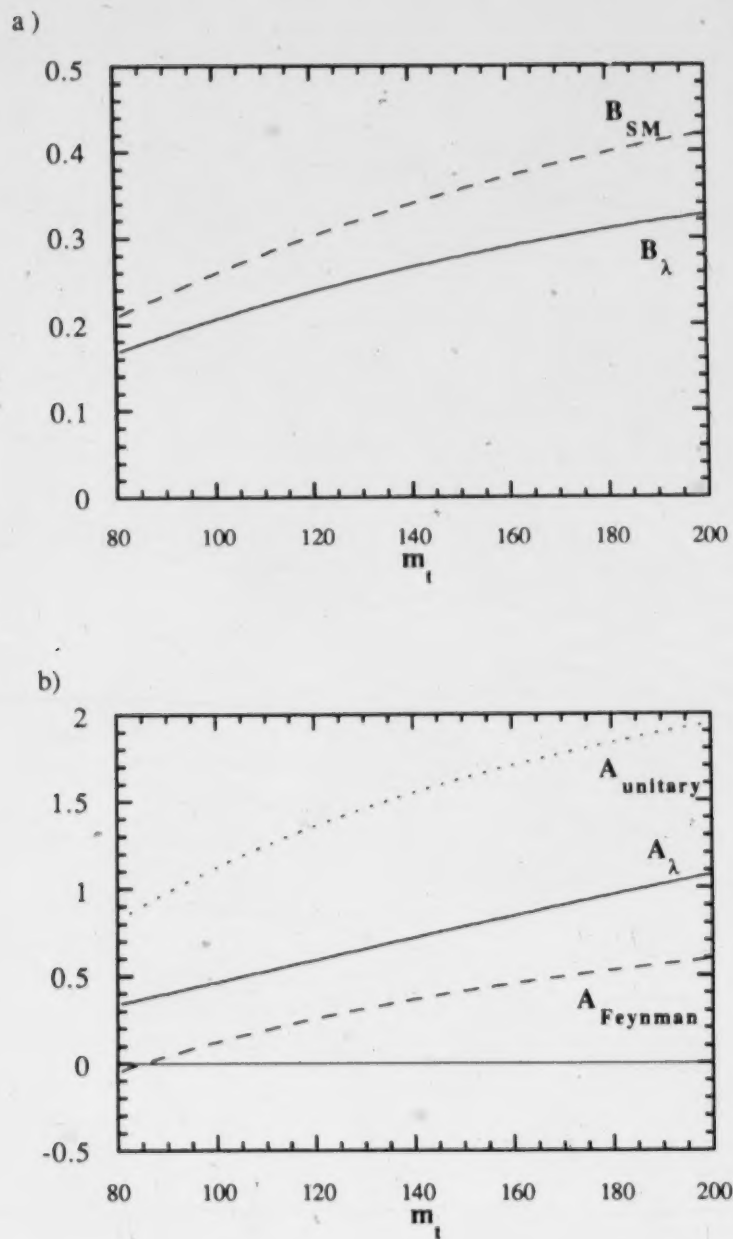


Figure 4.3: a) The value of B_λ (solid line) and B_{SM} (dashed line) as a function of top mass. b) The value of A_λ (solid line), A_{SM} in the unitary (dotted line) and A_{SM} in the Feynman (dashed line) gauges as functions of top mass.

For $b \rightarrow s\gamma$ only B contributes. Furthermore, since $B_{\text{tot}}(0) = 0$, one can effectively ignore the up and charm quark contributions and the vertex function for a real photon becomes

$$\Gamma_\mu(q) = \frac{-em_b G_F}{4\sqrt{2}\pi^2} K_{ts}^* K_{tb} B_{\text{tot}}(z_L) \sigma_{\mu\nu} q^\nu R, \quad (4.29)$$

where a small strange mass has also been ignored. As one can see from figure 4.3a, B_λ is of the same order as the standard model value, so we should expect to get reasonable results.

The decay rate for $b \rightarrow s\gamma$ is:

$$\Gamma(b \rightarrow s\gamma) = \frac{\alpha G_F^2}{128\pi^4} |K_{ts}^* K_{tb}|^2 m_b^5 |B_{\text{tot}}|^2. \quad (4.30)$$

As yet we have not included QCD corrections which can be added [Grin.88a, Grig.88] by changing B_{tot} to

$$B_{\text{tot}}^{\text{QCD}}(z_L) = a [B_{\text{tot}}(z_L) + b], \quad (4.31)$$

where

$$a = \eta^{-16/23}, \quad (4.32a)$$

$$b = \frac{116}{135} (\eta^{10/23} - 1) + \frac{58}{189} (\eta^{28/23} - 1), \quad (4.32b)$$

and $\eta = \alpha_s(m_b^2) / \alpha_s(m_W^2)$. The values of a and b for $\Lambda_{\text{QCD}} = 150$ MeV are $a = 0.65$ and $b = 0.33$. To compare the branching ratio of $b \rightarrow s\gamma$ with experiment requires some knowledge of the C-K-M and hadronic matrix elements. Some of this uncertainty can be done away with by taking the ratio of

$$\frac{\Gamma(b \rightarrow s\gamma)}{\Gamma(b \rightarrow c\bar{e}\nu)} = \frac{3\alpha}{2\pi} \frac{|B_{\text{tot}}^{\text{QCD}}|^2}{\rho(m_c/m_b)} \frac{|K_{ts}^* K_{tb}|^2}{|K_{cb}|^2}, \quad (4.33)$$

where, $\rho = 0.447$ is the phase space factor for $b \rightarrow c\bar{e}\nu$ with $m_b/m_c = 3$. One further assumes that the ratio of C-K-M matrix elements is approximately one.

We now will get a value for the branching ratio for the mesonic decay using [Riz.90]

$$\frac{\Gamma(\bar{B} \rightarrow K^* \gamma)}{\Gamma(\bar{B} \rightarrow \text{all})} = \frac{\Gamma(\bar{B} \rightarrow K^* \gamma)}{\Gamma(b \rightarrow s\gamma)} \frac{\Gamma(b \rightarrow s\gamma)}{\Gamma(b \rightarrow c\bar{e}\nu)} \frac{\Gamma(\bar{B} \rightarrow X_c \bar{e}\nu)}{\Gamma(\bar{B} \rightarrow \text{all})} = r_1 \cdot r_2 \cdot r_3. \quad (4.34)$$

r_3 is the particle data group value for the charm portion of the inclusive $\bar{B}$ goes to $e\nu_e$ decay: $r_3 = 0.113 \pm 0.005$ [PDG.90]. r_1 has been calculated theoretically by Deshpande et al [Des.87] in the constituent quark model. A value $r_1 = 0.06$ was obtained there. However, there may be a way to calculate this value from experimentally known $D \rightarrow K^*$ form factors using the work of Isgur and Wise [Isg.90].

The form factors for the hadronic decay $\bar{B} \rightarrow K^* \gamma$ can be parametrized [Isg.90] by

$$\begin{aligned} & \langle K^*(p_2, \epsilon) | \bar{s} \sigma_{\mu\nu} b | \bar{B}(p_1) \rangle \\ &= g_+ \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (p_1 + p_2)^\sigma + g_- \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (p_1 - p_2)^\sigma + h \epsilon_{\mu\nu\lambda\sigma} (p_1 + p_2)^\lambda (p_1 - p_2)^\sigma (\epsilon^* \cdot p), \\ & \langle K^*(p_2, \epsilon) | \bar{s} \sigma_{\mu\nu} \gamma_5 b | \bar{B}(p_1) \rangle \\ &= i g_+ [\epsilon_\nu^* (p_1 + p_2)_\mu - \epsilon_\mu^* (p_1 + p_2)_\nu] + i g_- [\epsilon_\nu^* (p_1 - p_2)_\mu - \epsilon_\mu^* (p_1 - p_2)_\nu] \\ &+ i h [(p_1 + p_2)_\nu (p_1 - p_2)_\mu - (p_1 - p_2)_\nu (p_1 + p_2)_\mu] (\epsilon^* \cdot p). \end{aligned} \quad (4.35)$$

Restricting this to a right-handed b and contracting it with q^ν , as one must do for $\bar{B} \rightarrow K^* \gamma$, this reduces to just one form factor:

$$\begin{aligned} & \langle K^*(p_2, \epsilon) | \bar{s} \sigma_{\mu\nu} \frac{1}{2} (1 + \gamma_5) b | \bar{B}(p_1) \rangle q^\nu \\ &= (g_+ (q^2)) (p_1 - p_2)^\mu \{ \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (p_1 + p_2)^\sigma + i [\epsilon_\nu^* (p_1 + p_2)_\mu - \epsilon_\mu^* (p_1 + p_2)_\nu] \}. \end{aligned} \quad (4.36)$$

This form factor can be related to the ratio of the decay rate $\Gamma(\bar{B} \rightarrow K^* \gamma)$ to the inclusive rate $\Gamma(b \rightarrow s \gamma)$ through [Des.87] (Deshpande et al's $f_1 = 2g_+$)

$$r_1 = \frac{\Gamma(\bar{B} \rightarrow K^* \gamma)}{\Gamma(b \rightarrow s \gamma)} = \left[\frac{m_b (m_B^2 - m_{K^*}^2)}{m_B (m_b^2 - m_s^2)} \right]^3 \left(1 + \frac{m_s^2}{m_b^2} \right)^{-1} (4 \cdot |g_+|^2). \quad (4.37)$$

It is now possible to relate the above form factors to those which occur in the vector and axial current,

$$\begin{aligned} & \langle K^*(\vec{p}_2, \vec{\epsilon}) | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}(\vec{p}_1) \rangle = f \epsilon_\mu^* + a_+ (\epsilon^* \cdot p) (p_1 + p_2)_\mu + a_- (\epsilon^* \cdot p) (p_1 - p_2)_\mu, \\ & \langle K^*(\vec{p}_2, \vec{\epsilon}) | \bar{s} \gamma_\mu b | \bar{B}(\vec{p}_1) \rangle = i g \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} (p_1 + p_2)^\lambda (p_1 - p_2)^\sigma. \end{aligned} \quad (4.38)$$

By looking at an effective theory in the rest frame of the $\bar{B}$ meson, one finds that equation (4.35) reduces to the spatial components of equation (4.38) leading to the following relation between g_+ and f and g :

$$g_+ = \frac{1}{2} \left[\frac{f}{2m_B} + \frac{p_1 \cdot p_2}{m_B} g - m_B g \right]. \quad (4.39)$$

There are also similar relations involving g_- and h and a_+ , a_- , f and g , but we are only

interested in g_+ .

We are now in a position to use Isgur and Wise's relations between the form factors for $\bar{B} \rightarrow K^*$ and $D \rightarrow K^*$. In an effective theory where both the charm and bottom quark are treated as heavy there is an SU(2) flavour symmetry [Vol.87, Pol.88, Isg.89, Eic.90, Geo.90]

$$\begin{pmatrix} c \\ b \end{pmatrix} \rightarrow U \begin{pmatrix} c \\ b \end{pmatrix}, \quad U \in \text{SU}(2). \quad (4.40)$$

This leads to the following relations [Isg.90]:

$$\begin{aligned} g(\bar{B} \rightarrow K^*) &= \left(\frac{m_c}{m_b}\right)^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} g(D \rightarrow K^*) \\ f(\bar{B} \rightarrow K^*) &= \left(\frac{m_b}{m_c}\right)^{1/2} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} f(D \rightarrow K^*) \end{aligned} \quad (4.41)$$

As a final step, we will require another parametrization of the form factors in the vector and axial vector currents of $D \rightarrow K^*$. This parametrization is [Bau.87, Gil.90]:

$$\begin{aligned} &\langle K^*(p_2, \epsilon) | \bar{s} \gamma_\mu \frac{1}{2}(1 - \gamma_5) b | D(p_1) \rangle \\ &= -(m_D + m_{K^*}) A_1(q^2) \epsilon_\mu^* - \frac{A_2(q^2)}{m_D + m_{K^*}} \epsilon^* \cdot q (p_1 + p_2)_\mu - \frac{i 2V(q^2)}{m_D + m_{K^*}} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p_1^\rho p_2^\sigma, \end{aligned} \quad (4.42)$$

where a fourth form factor, A_3 has been ignored in the limit of zero lepton mass. This requires

$$\begin{aligned} f(q^2) &= (m_D + m_{K^*}) A_1(q^2), \\ g(q^2) &= \frac{V(q^2)}{m_D + m_{K^*}}, \\ a_+(q^2) &= \frac{-A_2(q^2)}{m_D + m_{K^*}}, \end{aligned} \quad (4.44)$$

and a_- can be ignored in the limit of zero lepton mass. The Tagged Photon Spectrometer Collaboration [Anj.90] quote values for form factors in the decay $D \rightarrow K^{*0} e^+ \nu_e$ of $A_1(0) = 0.46 \pm 0.05 \pm 0.05$ ($f(0) = 1.27 \pm 0.14 \pm 0.14$ GeV) and $V(0) = 0.9 \pm 0.3 \pm 0.1$ ($g(0) = 0.32 \pm 0.11 \pm 0.04$ GeV⁻¹), where the values in brackets were obtained using equation (4.44) with $m_D = 1.87$ GeV and $m_{K^*} = 0.89$ GeV. Using the central values of these form factors and Isgur and Wise's relations (equation (4.41)), which should be valid even at $q^2=0$ [Isg.91], one finds that $f(\bar{B} \rightarrow K^*) = 2.44$ GeV and $g(\bar{B} \rightarrow K^*) = 0.21$ (GeV)⁻¹, for $m_b = 4.9$ GeV and $m_c = 1.63$ GeV (i.e. $m_b/m_c = 3$) and $\alpha_s(m_b)/\alpha_s(m_c) = 0.65$,

corresponding to $\Lambda_{\text{QCD}} = 200$ MeV for four flavours. Using equations (4.39) and (4.37) one finds that $g_+ = -0.15$ and $r_1 = 0.11$ when, in addition to the already quoted quark and meson masses, $m_B = 5.2$ GeV, and $m_s = 0.15$ GeV. The value $g_+ = -0.15$ should be compared with a value $g_+ = -0.114$ calculated by Deshpande et al in the constituent quark model [Des.87]. Considering only the range of values on the form factors, but keeping all the other parameters constant, the value of r_1 can range from $r_1 = 0.01$ to 0.31 . For the sake of argument, a value of $r_1 = 0.11$ will be used here.

We can now return to our calculation of our anomalous couplings on the branching ratios. Let's assume that the anomalous coupling dominates all other contributions, including the additive QCD correction, so that $B_{\text{tot}}^{\text{QCD}}$ is given by the B_λ given in equation (4.24) multiplied by the coupling λ and the multiplicative QCD correction, $\eta^{-16/23}$. Inserting this into equations (4.33) and (4.34) and demanding that the branching ratio must be below the experimentally determined bound, which is [PDG.90]

$$\frac{\Gamma(\bar{B} \rightarrow K^* \gamma)}{\Gamma(\bar{B} \rightarrow \text{all})} \leq 2.8 \times 10^{-4}, \quad (4.45)$$

one has that $\lambda \leq 2.5 / |B_\lambda|$ for $\alpha = 1/128$. Figure 4.4a shows the ratio $2.5 / |B_\lambda|$ as a function of top mass. For a top mass of 135 GeV, for which $B_\lambda = 0.26$, one has $|\lambda| \leq 10$. This compares well with the bounds from g-2 of the muon [Gra.85, Wal.85]. The next question one might ask is; how good a bound could one expect if the experimental values for $\bar{B} \rightarrow K^* \gamma$ were improved? This time not ignoring the Standard Model contribution, or the additive QCD correction, which for realistic λ_γ values would dominate $B_{\text{tot}}^{\text{QCD}}$, one can use equations (4.33) and (4.34) to obtain a branching rate for the process for various λ_γ . Figure 4.4b gives a plot of the branching rate of $\bar{B} \rightarrow K^* \gamma$ as a function of top mass for various λ_γ . Given improved bounds on $\bar{B} \rightarrow K^* \gamma$, one could expect to observe λ_γ of order one. This is to be compared with Chia's results, where $b \rightarrow s \gamma$ can also constrain the anomalous magnetic moment of the W, $\Delta\kappa$, to be of order one [Chi.90, Riz.90].

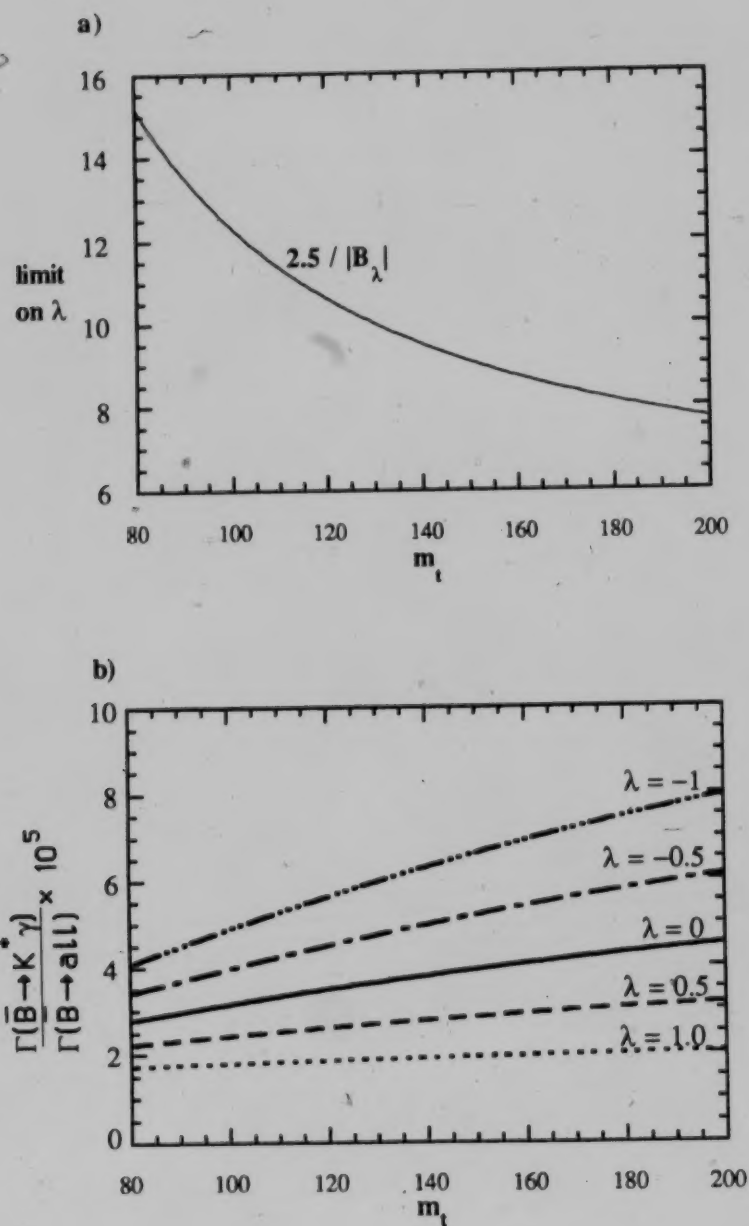


Figure 4.4: a) Limits on the $WW\gamma$ coupling, λ_γ , from experimental limits on $\bar{B} \rightarrow K^* \gamma$.

b) The branching ratio $\Gamma(\bar{B} \rightarrow K^* \gamma) / \Gamma(\bar{B} \rightarrow \text{all})$ as a function of top mass for various values of the $WW\gamma$ coupling, λ_γ .

4.5 Anomalous Contributions to $b \rightarrow s\mu^+\mu^-$

We now turn to $b \rightarrow s\mu^+\mu^-$ to see if anomalous couplings might have a noticeable effect there as well. Since $b \rightarrow s\mu^+\mu^-$ involves a virtual photon, it includes both A and B of equations (4.22) and (4.25). The decay, $b \rightarrow s\mu^+\mu^-$, can proceed through a W box diagram, through Z exchange, or through photon exchange. Only Z exchange and photon exchange can have contributions from the anomalous W couplings being discussed here. Indeed, we only need worry about contributions to photon exchange. For Z exchange, one gets identical results for the $b \rightarrow sZ^*$ vertex as one obtained for the virtual photon vertex in equation (4.22). However, in the case of the Z exchange, these are multiplied by the Z propagator $\sim (q^2 - M_Z^2)^{-1} = M_Z^{-2}$, which is small. The photon propagator $\sim q^{-2}$ produces a much larger result.

Again it is convenient to take the ratio of $\Gamma(b \rightarrow s\mu^+\mu^-)$ to $\Gamma(b \rightarrow c\bar{e}\nu)$ in order to alleviate some of the uncertainty in C-K-M and hadronic matrix elements. This ratio is given as [Grig.89, Grin.89]:

$$\frac{1}{\Gamma(\bar{B} \rightarrow X_{c\bar{e}\nu})} \frac{d\Gamma(\bar{B} \rightarrow X_{s\mu^+\mu^-})}{dx} = \frac{\alpha^2}{4\pi^2} \frac{1}{\rho(m_c/m_b)} (1-x)^2 [(1+2x)(|A + \frac{1}{2}C|^2 + \frac{1}{4}|C|^2) + (1+\frac{2}{x})|B|^2 - 6\text{Re}(AB)], \quad (4.46)$$

where $x \equiv q^2 / m_b^2$. A is a combination of the $A(z_i)$ for the top and charm contributions to the virtual photon, and a contribution from Z exchange:

$$A = A_{\text{tot}}(x_t) - A_c + A_Z + \frac{4}{9}. \quad (4.47)$$

The term 4/9 in (26) is in order to bring it in line with the renormalization scheme used in [Grin.89], it is partly compensated by a term of 4/9 in A_c . B comes completely from the real photon contribution:

$$B = \sum_j B_{\text{tot}}(x_j) = B_{\text{tot}}(x_t). \quad (4.48)$$

Finally C comes from the W box diagrams and the portion of Z exchange that is totally left-handed. The values for these various parameters are:

$$A_c = \frac{4}{9} \left[\ln \frac{m_c^2}{\mu^2} + f\left(\frac{q^2}{4m_c^2}\right) \right] \left[\frac{1}{2} (\eta^{-6/23} + \eta^{12/23}) + \frac{3}{2} (\eta^{-6/23} - \eta^{12/23}) \right], \quad (4.49)$$

where

$$f(y) = -\left(\frac{2}{3} + \frac{1}{y}\right) + [\theta(y) - \theta(y-1)] \cdot 2(1 + \frac{2}{y})(\frac{1}{y} - 1)^{1/2} \tan^{-1}[(\frac{1}{y} - 1)^{1/2}] \\ + \theta(y-1) (1 + \frac{1}{2y})(1 - \frac{1}{y})^{1/2} \left[\ln\left(\frac{1 + (1 - \frac{1}{y})^{1/2}}{1 - (1 - \frac{1}{y})^{1/2}}\right) - i\pi \right]. \quad (4.50)$$

The parameter A_Z is gauge dependent, in the Feynman gauge

$$A_Z = z_t \left(\frac{z_t - 6}{2(1 - z_t)} - \frac{z_t + 2}{2(1 - z_t)^2} \ln z_t \right), \quad (4.51)$$

and

$$C = \frac{-1}{\sin^2 \theta_W} \left[\frac{A_Z}{2} + \frac{z_t}{2} \left(\frac{1}{1 - z_t} + \frac{\ln z_t}{(1 - z_t)^2} \right) \right]. \quad (4.52)$$

Finally, A_{tot} and B_{tot} must be their QCD corrected values,

$$A_{\text{tot}}^{\text{QCD}} = A_{\text{tot}} + \left[\frac{4\pi}{\alpha_s(M_W)} \right] \left[\frac{8}{87} (1 - \eta^{-29/23}) - \frac{4}{33} (1 - \eta^{-11/23}) \right], \quad (4.53)$$

and the QCD corrected B value was given earlier by equation (4.31).

One does not expect to improve on the bound on λ_γ from $b \rightarrow s\gamma$, given that the bounds on $b \rightarrow s\mu^+\mu^-$ are of the same order as bounds on $b \rightarrow s\gamma$. Therefore, we will proceed right to the question of whether a λ_γ of order one could effect the theoretical branching ratio. Figure 4.5 is a graph of the integrated rate $\Gamma(b \rightarrow s\mu^+\mu^-) / \Gamma(b \rightarrow ce\nu)$ as a function of top mass for various λ_γ . It was calculated using $\Lambda_{\text{QCD}} = 150$ MeV, $m_c = 1.63$ GeV, $m_b = 4.9$ GeV, and $m_s = m_\mu = 0$, except that the integrated rate was cutoff at $x = 4m_\mu^2 / m_b^2$. Once again an anomalous quadrupole coupling of order one can clearly effect the branching rate. It should be noted that the same differential rate would apply to $b \rightarrow se^+e^-$, however, one would undoubtedly wish to choose a more realistic value for the low x cutoff in the integrated rate.

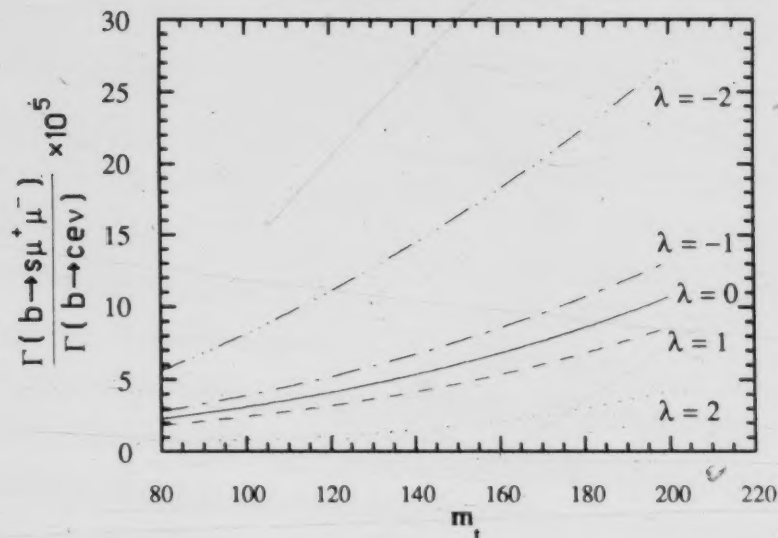


Figure 4.5: The ratio $10^5 \times \Gamma(b \rightarrow s \mu^+ \mu^-) / \Gamma(b \rightarrow c e \nu)$ as a function of top mass for various values of the $WW\gamma$ coupling, λ_γ .

4.6 Concluding Remarks

To sum up, the current bounds on $\bar{B} \rightarrow K^* \gamma$, for which $\text{BR}(\bar{B} \rightarrow K^* \gamma) \leq 2.8 \times 10^{-4}$, can constrain a $WW\gamma$ anomalous quadrupole coupling to be of order 10 as is demonstrated in figure 4.4a. In particular, for a top mass of 135 GeV, one has $\lambda \leq 10$. Soon to be expected constraints from CLEO should constrain this even better. This compares well with constraints from $g-2$ of the muon. Furthermore, improved bounds on the branching rates for $\bar{B} \rightarrow K^* \gamma$ and $\bar{B} \rightarrow X_s \mu^+ \mu^-$ could constrain the coupling to order one as is illustrated in figures 4.4b and 4.5 respectively. This, however, assumes that QCD corrections and the ratio of exclusive to inclusive $b \rightarrow s$ decays are well known.

5. THE FINAL CHAPTER

In the introduction to this thesis we reviewed how flavour-changing interactions and other rare processes have served as a good test of the Standard Model. Moreover, they have also put stringent constraints on any new physics beyond the Standard Model. Specifically, it was mentioned how these processes constrained both supersymmetric theories and theories evoking a new level of substructure in the elementary particles (composite models, including Technicolour). The bulk of this thesis was devoted to seeing how these same processes would constrain possible models with both a composite and supersymmetric nature.

Unlike a purely composite model, where the composite nature would induce effective low energy Lagrangians for these processes that are suppressed by two powers of the composite mass scale, the effective low energy Lagrangians induced by supercomposite models would be suppressed by one power each of the composite mass scale and the supersymmetric mass scale. If the supersymmetric mass scale is much lower than the composite mass scale, then these will dominate over interactions induced by the purely composite nature of the theory. As such, for a given scale of supersymmetry breaking, one can set limits on the scale of compositeness by imposing that these models cannot contribute to various rare processes at rates contradicting with experiment.

We calculated what effect a supercomposite model might have on four processes: the decay of the pion into two leptons (π_{2l} decay), muon conversion ($\mu N \rightarrow e N$), the K_L - K_S mass difference (and also B - $\bar{B}$ mixing) and finally to the neutron's electric dipole moment. From π_{2l} decay we found that one could constrain the composite mass scale to

be 980 TeV (98 TeV) for a supersymmetric theory with a supersymmetric mass scale of 100 GeV (1 TeV). This is to be compared to a limit of 200 TeV that could be set from a purely composite model, although it would also apply to supercomposite models if the supersymmetric mass scale was so high as to eliminate any of the advantages of dressing two-fermion, two-scalar interactions to four-fermion interactions with the exchange of supersymmetric gauginos. This decay tests the compositeness of quarks and leptons, where the compositeness does not lead to flavour changing interactions. One might expect this type of compositeness if each generation of quarks and leptons is built out of different constituents.

From π_{21} decay we moved onto processes that did not respect the flavour conservation laws of the Standard Model. Considering a lepton flavour-violating process, we find that the process of the conversion of a muon into a electron in the presence of a nucleon can constrain the supercomposite mass scale to be greater than 6×10^4 TeV (6×10^3 TeV) for supersymmetric masses of order 100 GeV (1 TeV). This is considerably greater than the limit of 650 TeV that can be imposed in a purely composite model. These constraints would be imposed on composite models of the leptons (and quarks), where the various generations of leptons were somehow composed of the same constituents. In such a case one would expect the effective Lagrangians not to respect Standard Model conservation laws.

For our final two-scalar, two-fermion interaction dressed to a four-fermion interaction, we considered the K_L - K_S mass difference. Again this is a process that involves composite quarks that do not obey Standard Model conservation laws. Because of chiral constraints, we found that this decay was suppressed by two powers of small quark masses (divided by weak scale masses). Thus the constraints on the supercomposite mass scale we found from this process were significantly lower than what

could be obtained via a pure composite process, as we obtained a constraint of only 3-11 TeV for supersymmetric masses of order the weak scale in the supercomposite case, and a constraint of 2,500 TeV from a purely composite interaction. These constraints could be applied to composite models of quarks where the various generations of quarks are comprised of the same constituents. The effect of supercomposite interactions on $B-\bar{B}$ was also investigated. Here one is dealing with the large top mass and therefore the chiral constraint is not as important. Using the constraint on the composite mass of 2,500 TeV obtained from the K_L-K_S mass difference and a supersymmetric mass scale of 100 GeV, one finds that the supercomposite and composite contributions to $B-\bar{B}$ mixing are well below the experimentally observed value. Furthermore, the mixing goes as the third power of the supersymmetric mass scale. On a more positive note, the supercomposite process could be significant if the composite mass scale constraint of 2,500 TeV did not apply here, which might be the case if compositeness only manifested itself in the third generation of quarks.

As our last rare process for considering possible effects of supercomposite models we look at the neutron's electric dipole moment. Considering possible contributions to Weinberg's six gluon operator, the quark colour dipole moment, and the quark electric dipole moment, we found, for a supersymmetry mass scale of order 100 GeV, a supercomposite mass scale constraint of 1,000 TeV from a CP-violating form factor in the quark-squark-gluino coupling and a constraint of 20,000 TeV from a possible composite induced gluino colour dipole moment. However, this last constraint from the gluino colour dipole term would undoubtedly rely on the gluino being composite. Since the gluon cannot be composite [Gas.62, Wei.80], this probably implies that if this term exists, the supersymmetric mass scale would be of the same order as the composite mass scale. Considering this possibility we obtain constraints of 10 TeV for the CP-violating form

factors and 50 TeV for the gluon colour dipole moment. This is compared to constraints of the order of 130 TeV that can be obtained considering only composite processes. The purely composite constraints plus the constraints due to the CP-violating form factor in the gluino coupling could be attributed to composite models of quarks where the compositeness induced CP-violating effects. This would be a type of (super)compositeness quite similar to the type we considered in the K_L - K_S mass difference. The constraint obtained from the composite induced gluino colour dipole term could be attributed to a supercomposite model of the gaugino, but not of the gauge bosons. This is a type of supercompositeness that has not been widely investigated. A summary of all the constraints obtained on the scales of compositeness in pure composite models and the product of the scales of compositeness and supersymmetry in supercomposite models can be found in figure 3.4 to be found on page 74.

In the final analysis, we have considered a whole spectrum of possible supercomposite contributions to rare processes. We have considered non-flavour-changing interactions with π_{2I} decay. We have considered lepton and quark flavour-changing interactions with the investigations of $\mu N \rightarrow e N$ and the K_L - K_S mass difference respectively. Finally, the neutron electric dipole moment investigated possible CP-violating supercomposite decays.

The final piece of this thesis consisted of finding the effects of anomalous $WW\gamma$ couplings on $b \rightarrow s$ electromagnetic transitions. Specifically, we investigated the possible effects of an anomalous quadrupole coupling, λ_γ . From $b \rightarrow s\gamma$ transitions (or its mesonic equivalent, $\bar{B} \rightarrow K^*\gamma$) we found that λ_γ could be constrained to be of order 10 from present experimental bounds, and with improved bounds on the process, λ_γ could be constrained to be of order 1.

From a more general context, it would appear flavour-changing interactions and

other rare processes will continue to serve as stringent tests of the Standard Model and all its extensions or replacements. High precision tests have already proven their worth, and more of such tests with higher precision would be most welcome. It is expected that supersymmetry and composite models will both lead to some form of lepton number violation, which in the former case is fairly well documented. As such, flavour-changing interactions may well turn out to provide us with the first conclusive evidence of physics beyond the Standard Model. If not, then, as in the past, they will show us just how effective the Standard Model is as a theory, albeit a non-unified one, of the fundamental forces.

REFERENCES

- [Adl.69] S.L. Adler, Phys. Rev. **177** (1969), 2426; S.L. Adler, in *Lectures on Elementary Particle Physics and Quantum Field Theory*, S. Deser, M. Grisaru, and H. Pendleton, eds. (M.I.T. Press, Cambridge, 1970), 1.
- [Alt.86] I.S. Altarev et al, Pis'ma Zh. Eksp. Teor. Fiz. **44** (1986), 360 [JETP Lett. **44**, 460 (1986)]
- [Alt.88] T. Altomari, Phys. Rev. **D37** (1988), 677.
- [Ama.91] U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. **260B** (1991), 447.
- [And.63] P.W. Anderson, Phys. Rev. **130** (1963), 439.
- [Anj.90] J.C. Anjos et al., Phys. Rev. Lett. **65** (1990), 2630.
- [ARG.87] ARGUS Collaboration, H. Albrecht et al., Phys. Lett. **192B** (1987), 245.
- [ARG.89] ARGUS Collaboration, H. Albrecht et al., Phys. Lett. **B229** (1989), 304.
- [Arn.90] R. Arnowitt, J.L. Lopez and D.V. Nanopoulos, Phys. Rev. **D42** (1990), 2423.
- [Arn.91] R. Arnowitt, M.J. Duff and K.S. Stelle, Phys. Rev. **D43** (1991), 3085.
- [Ave.89] P. Avery et al., Phys. Lett. **B223** (1989), 470.
- [Bai.77] D. Bailin, *Weak Interactions* (Adam Hilger Ltd., Bristol, 1977).
- [Bar.82] R. Barbieri and L. Maiani, Phys. Lett. **117B** (1982), 203.
- [Bar.83] R. Barbieri, A. Masiero, and G. Veneziano, Phys. Lett. **128B** (1983), 179.
- [Bau.87] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. **C40** (1987), 103.
- [Bau.88a] U. Baur and D. Zeppenfeld, Phys. Lett. **201B** (1988), 383.
- [Bau.88b] U. Baur and D. Zeppenfeld, Nucl. Phys. **B308** (1988), 127.

- [Bau.90] U. Baur and E.L. Berger, Phys. Rev. **D41** (1990), 1476.
- [Bel.69] J.S. Bell and R. Jackiw, Nuov. Cim. **60A** (1969), 47.
- [Bel.90] G. Bélanger, F. Boudjema and D. London, Phys. Rev. **D65**, (1990) 2943.
- [Ber.87a] S. Bertolini, F. Borzumati and A. Masiero, Phys. Rev. Lett. **59**, (1987) 180.
- [Ber.87b] S. Bertolini, F. Borzumati and A. Masiero, Phys. Lett. **192B**, (1987) 119.
- [Bjo.64] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, Inc., New York, 1964).
- [Bjo.79] J.D. Bjorken, Phys. Rev. **D19** (1979), 335.
- [Boy.90] G. Boyd, A.K. Gupta, S.P. Trivedi and M.B. Wise, Phys. Lett. **241B** (1990), 584.
- [Bra.90] E. Braaten, C.S. Li and T.C. Yuan, Phys. Rev. **D64** (1990) 1709.
- [Bre.69] M. Breidenbach, J.I. Friedman, H.W. Kendall, E.D. Bloom, D.H. Coward, H. DeSraebler, J. Drees, L.W. Mo, and R.E. Taylor, Phys. Rev. Lett. **23** (1969), 935.
- [Buc.83] W. Buchmüller, R.D. Peccei and T. Yanagida, Phys. Lett. **124B** (1983), 67; Nucl. Phys. **227** (1983), 503; Nucl. Phys. **231** (1984), 53; Nucl. Phys. **244** (1984), 186.
- [Cam.82] B.A. Campbell and P.J. O'Donnell, Phys. Rev. **D25**, (1982) 1989.
- [Cam.84] B.A. Campbell and J.A. Scott, Phys. Lett. **135B** (1984), 423.
- [Cam.90] B.A. Campbell, "Beyond the Standard Model", lectures presented at the Fifth Lake Louise Winter Institute - *The Standard Model and Beyond*, World Scientific, Singapore (1990), 138.
- [Cas.62] K.M. Case and S.G. Gasiorowicz, Phys. Rev. **125** (1962) 1055.
- [Cha.90] Darwin Chang, Chong Sheng Li and Tzu Chiang Yuan, Phys. Rev. **D42** (1990), 867.

- [Che.77] T.P. Cheng and L.F. Li, Phys. Rev. **D16** (1977), 1425.
- [Chi.90] S.P. Chia, Phys. Lett. **240B**, (1990) 465.
- [Col.67] S. Coleman and J. Mandula, Phys. Rev. **159** (1967), 1251.
- [Cou.91] G. Couture, S. Godfrey, and R. Lewis, OCIP/C 91-3.
- [Dai.90a] J. Dai, H. Dykstra, R.G. Leigh, S. Paban, and D.A. Dicus, Phys. Lett. **237B** (1990), 216.
- [Dai.90b] J. Dai and H. Dykstra, Phys Lett. **237B** (1990) 256.
- [Dan.62] G. Danby, J.-M. Gaillard, K. Goulianos, L.M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Lett. **9** (1962), 36.
- [Des.87] N.G. Deshpande et al., Phys. Rev. Lett. **59** (1987), 183.
- [Des.88] N.G. Deshpande, P. Lo, and J. Trampetic, Z. Phys. **C40** (1988), 40.
- [DeW.75] B. de Wit and D.Z. Freedman, Phys. Rev. **D12** (1975), 2286.
- [Din.90] M. Dine and W Fischler, Phys. Lett. **242B** (1990), 239.
- [Don.79] J.F. Donoghue and L.-F. Li, Phys. Rev. **D19** (1979), 945.
- [Eic.90] E. Eichten and B. Hill, Phys. Lett. **234B** (1990), 511.
- [Ein.05] A. Einstein, "Zur Elektrodynamik Bewegter Körper," Ann. Phys. (Germany) **17** (1905), 891; or for an English Translation: A. Einstein, H.A. Lorentz, H. Minkowski, and H. Weyl, *The Principle of Relativity*, Mathuen & Co. Ltd. (1923), 36.
- [Ell.82] J. Ellis, J. Hagelin, and D.V. Nanopoulos, Phys. Lett. **116B** (1982), 283.
- [Ell.83] J. Ellis, in *Dynamics and Spectroscopy at High Energies*, proceedings of the SLAC Summer Institute, P.M. McDonough ed. (1983), 239.
- [Ell.86] R.G. Ellis, G.C. Joshi and M. Matsuda, Phys. Lett. **179B**, (1986) 119.
- [Eng.64] F. Englert and R. Brout, Phys. Rev. Lett. **13** (1964), 321.

- [Fay.80] P. Fayet, in *Unification of the Fundamental Particle Interactions* (Plenum, New York, 1986), 587.
- [Fra.90] M. Franklin, "Physics at the Proton Anti-Proton Colliders," lectures presented at the Fifth Lake Louise Winter Institute - *The Standard Model and Beyond*, World Scientific, Singapore (1990), 65.
- [Gai.74] M.K. Gaillard and B.W. Lee, Phys. Rev. **D10** (1974), 897.
- [Gai.75] M.K. Gaillard, B.W. Lee, and J.L. Rosner, Rev. Mod. Phys. **47** (1975), 277.
- [Gel.64] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way*, (Benjamin, W.A. New York, 1964); M. Gell-Mann, Phys. Lett. **8** (1964), 214.
- [Geo.74] H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32** (1974), 438.
- [Geo.82] H. Georgi, *Lie Algebras in Particle Physics*, (Benjamin/Cummings Publishing Company, Menlo Park, California, 1982).
- [Geo.86] H. Georgi and L. Randall, Nucl. Phys. **B276** (1986) 241.
- [Geo.90] H. Georgi, Phys. Lett. **240B** (1990), 447.
- [Gil.90] F.J. Gilman and R.L. Singleton Jr., Phys. Rev. **D41** (1990), 142
- [Gla.61] S.L. Glashow, Nucl. Phys. **22** (1961), 579.
- [Gla.70] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. **D2** (1970), 1285.
- [God.91] S. Godfrey, OCIP/C 91-2 (1991).
- [Gol.61] J. Goldstone, Nuovo Cimento **19** (1961), 154.]
- [Gra.85] A. Grau and J.A. Grifols, Phys. Lett. **154B**, (1985) 283.
- [Grif.88] J.A. Grifols, S. Peris and J. Solá, Int. J. of Mod. Phys., **A3**, (1988) 225.
- [Grig.88] R. Grigjanis, P.J. O'Donnell, M. Sutherland and H. Navelet, Phys. Lett. **213B**, (1988) 355.
- [Grig.89] R. Grigjanis, P.J. O'Donnell, M. Sutherland and H. Navelet, Phys. Lett.

223B, (1989) 239.

- [Grin.88a] B. Grinstein and M.B. Wise, Phys. Lett. **201B**, (1988) 274.
- [Grin.88b] B. Grinstein, R. Springer and M.B. Wise, Phys. Lett. **202B**, (1988) 138.
- [Grin.89] B. Grinstein, M.J. Savage and M.B. Wise, Nucl. Phys. **B319**, (1989) 271.
- [Gur.64] G.S. Guralnik, C.R. Hagen, and T.W.B. Kibble, Phys. Rev. Lett. **13** (1964), 585.
- [Hab.79] H.E. Haber, G.L. Kane and T. Sterling, Nucl. Phys. **B161** (1979), 493.
- [Hab.85] H.E. Haber and G.L. Kane, Phys. Rep. **117** (1985), 75.
- [Hal.86] L.J. Hall, V.A. Kostelecky, and S. Raby, Nucl. Phys. **B267** (1986), 415.
- [Hag.87a] K. Hagiwara, Proceedings of the 2nd International Conference on Hadron Spectroscopy (Tsukuba, Japan), report KEK 87-7 (National Laboratory for High Energy Physics, Tsukuba, Japan, 1987), 315.
- [Hag.87b] K. Hagiwara, R.D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. **B282**, (1987) 253.
- [Haw.80] S.W. Hawking, D.N. Page, and C.N. Pope, Nucl. Phys. **B170** (1980), 283.
- [Hei.32] W. Heisenberg, Zeitschrift für Physik, **77** (1932), 1.
- [Hew.87] J.L. Hewett, Phys. Lett. **193B**, (1987) 327.
- [Hig.64] P.W. Higgs, Phys. Lett. **12** (1964), 132; Phys. Lett. **13** (1964), 508; Phys. Rev. **145** (1966), 1156.
- [Hou.87] W.S. Hou, A. Soni and H. Steger, Phys. Lett. **192B**, (1987) 441.
- [Hou.88] W.S. Hou and R.S. Willey, Phys. Lett. **202B**, (1988) 591.
- [Hun.78] P.Q. Hung and J.J. Sakurai, Nucl. Phys. **B143** (1978), 81.
- [Isg.89] N. Isgur and M.B. Wise, Phys. Lett. **232B** (1989), 113; Phys. Lett. **237B** (1990), 527.

- [Isg.90] N. Isgur and M.B. Wise, Phys. Rev. D**42** (1990), 2388.
- [Isg.91] N. Isgur, Phys. Rev. D**43** (1991), 810.
- [Ina.81] T. Inami and C.S. Lim, Prog. Theor. Phys. **65**, (1981) 297.
- [Jog.84] S.D. Joglekar, in *Supersymmetry and Supergravity Nonperturbative QCD*, Proceedings of Winter School held in Mahabeleshwar, India, Jan. 5-19, 1984, eds. P. Roy and V. Singh, in Lecture Notes in Physics #**208**.
- [Kan.89] G.L. Kane, J. Vidal and C.P. Yuan, Phys. Rev. D**39**, (1989) 2617.
- [Kib.67] T.W.B. Kibble, Phys. Rev. **155** (1967), 1554.
- [Kob.73] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. (Kyoto) **49** (1973), 652.
- [Kro.64] R. Kronig and V.F. Weisskopf, eds., *Collected Scientific Papers by Wolfgang Pauli*, Vol. II (Wiley-Interscience, New York, 1964), 1316.
- [Kur.87] M. Kuroda, F.M. Renard and D. Schildknect, Phys. Lett. **183B**, (1987) 366.
- [Man.84] A. Manohar and H. Georgi, Nucl. Phys. **B234** (1984) 189.
- [McW.80] B. McWilliams and O. Shankar, Phys. Rev. D**22** (1980), 2853.
- [McW.81] B. McWilliams and L.-F. Li, Nucl. Phys. **B179** (1981), 62.
- [Moh.86] R.N. Mohaptra, *Unification and Supersymmetry* (Springer-Verlag, New York, 1986).
- [Noe.18] E. Noether, *Nachrichten Gessel. Wissenschaft. Gottingen* **2** (1918), 235.
- [Nam.61] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961), 345.
- [O'Do.86] P.J. O'Donnell, Phys. Lett. **175B** (1986), 369.
- [Par.85] H.S. Park et al, Phys. Rev. Lett. **54** (1985), 22.
- [Pat.73] J.C. Pati and A. Salam, Phys. Rev. D**8** (1973), 1240.
- [PDG.90] Particle Data Group, Phys. Lett. **239B** (1990), 1.

- [Pec.87] R. Peccei, *Selected Topics in Electroweak Interactions*, *Proc. 2nd Lake Louise Winter Institute*, eds. J.M. Cameron et al. (World Scientific, Singapore, 1987).
- [Per.87] D.H. Perkins, *Introduction to High Energy Physics* (Addison-Wesley Publishing Company, Inc., Menlo Park, California, 1987).
- [Pes.81] M. Peskin, *Proc. Intern. Symp. on Lepton and Photon Interactions at High Energy* (Kyoto, 1985).
- [Pol.88] H.D. Politser and M.B. Wise, *Phys. Lett.* **206B** (1988), 681; *Phys. Lett.* **208B** (1988), 504.
- [Riz.88] T.G. Rizzo, *Phys. Rev.* **D38** (1988), 820.
- [Riz.90] T.G. Rizzo, MAD/PH/593.
- [Ros.85] G. Ross, *Grand Unified Theories* (Benjamin, New York, 1985).
- [Sak.82] N. Sakai and T. Yanagida, *Nucl. Phys.* **B197** (1982), 533.
- [Sal.64] A. Salam and J.C. Ward, *Phys. Lett.* **13** (1964), 168.
- [Sal.68] A. Salam, in *Elementary Particle Physics* (Nobel Symposium No. 8), ed. by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), 367.
- [Sha.55] R. Shaw, Cambridge Ph. D. Thesis (1955), unpublished.
- [Sha.79] O. Shankar, *Phys. Rev.* **D20** (1979), 1608.
- [Sha.82] O. Shankar, *Nucl. Phys.* **B204** (1982), 375.
- [Sim.85] J. Simpson, *Phys. Rev. Lett.* **54** (1985), 1891.
- [Sim.89] J. Simpson and A. Hime,* *Phys. Rev.* **D39** (1989), 1825; *Phys. Rev.* **D39** (1989), 1837.
- [Smi.90] K.F. Smith et al, *Phys. Lett.* **234B** (1990), 191.
- [tHo.71] G. 't Hooft, *Nucl Phys.* **B33** (1971), 173; *Nucl. Phys.* **B35** (1971), 167.

- ['tHo.80] G. 't Hooft, in *Recent Developments in Gauge Theories*, G. 't Hooft, C. Itzykson, A. Jaffe, H. Lehmann, P.K. Mitter, I.M. Singer, and R. Stora, eds. (Plenum Press, N.Y., 1980), 135.
- [UA1.83] UA1 Collaboration, G. Arnison et al., Phys. Lett. **122B** (1983), 95; **126B** (1983), 398; **129B** (1983), 273.
- [UA2.83] UA2 Collaboration, M. Banner et al., Phys. Lett. **122B** (1983), 476; **129B** (1983), 130.
- [Vol.87] M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45** (1987), 463 [Sov. J. Nucl. Phys. **45** (1987), 292].
- [Wal.85] J.C. Wallet, Phys. Rev. **D32** (1985), 813.
- [Wei.67] S. Weinberg, Phys. Rev. Lett. **19** (1967), 1264.
- [Wei.80] S. Weinberg and E. Witten, Phys. Lett. **96B** (1980), 59.
- [Wei.82] S. Weinberg, Phys. Rev. **D26** (1982), 287.
- [Wei.89] S. Weinberg, Phys. Rev. Lett. **63** (1989), 2333.
- [Wes.74] J. Wess and B. Zumino, Nucl. Phys. **B78** (1974), 1.
- [Yan.54] C.N. Yang and R.L. Mills, Phys. Rev. **96** (1954), 191.
- [Yuk.35] H. Yukawa, Proc. Phys. Math Soc. Japan **17** (1935), 48.
- [Zwe.64] G. Zweig, CERN Report 8182/TH401 (1964) unpublished; CERN Report 8419/TH412 (1964) unpublished.

APPENDIX A - SPINOROLOGY

In this thesis we will adopt the conventions of Bjorken and Drell [Bjo.64, Bjo.65] for our relativistic spinors. A Dirac spinor satisfies the Dirac equation:

$$(i\partial\!\!\!/ - m)\psi = 0, \quad \text{A.1}$$

where $\not{A} = a_\mu \gamma^\mu$ with the Minkowski metric (1, -1, -1, -1) and γ^μ are the 4x4 Dirac matrices, given by:

$$\gamma^0 = \begin{bmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{bmatrix}, \quad \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}, \quad \text{A.2}$$

with σ^i being the familiar 2x2 Pauli matrices and $\mathbf{1}$ a 2x2 unit matrix,

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \mathbf{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad \text{A.3}$$

The Pauli adjoint of ψ is given by $\bar{\psi} = \psi^\dagger \gamma^0$. Two other important Dirac operators are:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5 = \begin{bmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{bmatrix}, \quad \text{A.4}$$

and

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]; \quad \sigma^{ij} = \begin{bmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{bmatrix}, \text{ ijk cyclic, } \sigma^{0i} = i \begin{bmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{bmatrix}. \quad \text{A.5}$$

γ^5 anticommutes with all the γ^μ , while $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbf{1}$. $[a, b] = ab - ba$ represents the commutator, while $\{a, b\} = ab + ba$ represents the anticommutator.

One can now define projection operators,

$$L = \frac{1 - \gamma^5}{2}, \text{ and } R = \frac{1 + \gamma^5}{2}, \quad \text{A.6}$$

allowing the formation of chiral or Weyl fermions:

$$\psi_L = L\psi \quad (\bar{\psi}_L = \bar{\psi}R), \text{ and } \psi_R = R\psi \quad (\bar{\psi}_R = \bar{\psi}L). \quad \text{A.7}$$

One can easily see that

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L. \quad \text{A.8}$$

and

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R, \quad \text{A.9}$$

from which it is obvious that scalar or mass terms flip chirality and vector or gauge terms conserve chirality.

One final important concept concerning spinors is charge conjugation. The charge conjugate field is defined by:

$$\psi^c = C(\bar{\psi})^T \text{ and } \bar{\psi}^c = -\psi^T C^{-1}, \quad \text{A.10}$$

where C , the charge conjugation matrix is defined such that

$$C\gamma^\mu C^{-1} = -(\gamma^\mu)^T. \quad \text{A.11}$$

In the representation used in this thesis

$$C = -C^{-1} = -C^T = -C^\dagger = i\gamma^2\gamma^0 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \quad \text{A.12}$$

For chiral spinors, one has:

$$\psi_{L,R}^c = C(\bar{\psi}_{R,L})^T \quad (\bar{\psi}_{L,R}^c = -(\psi_{R,L})^T C^{-1}), \quad \text{A.13}$$

and

$$\psi_{L,R} = C(\bar{\psi}_{R,L}^c)^T \quad (\bar{\psi}_{L,R} = -(\psi_{R,L}^c)^T C^{-1}), \quad \text{A.14}$$

the last equation showing that the inverse relations are the same as the defining ones. Acting on the various Dirac operators gives:

$$CO_i C^{-1} = -(O_i)^T \quad \text{A.15a}$$

for $O_i = \gamma^\mu, \sigma^{\mu\nu}$, and

$$CO_i C^{-1} = (O_i)^T \quad \text{A.15b}$$

for $O_i = 1, \gamma^5, \gamma^\mu\gamma^5$.

With the definition of charge conjugation, we are now able to define Majorana spinors. They are defined by $\eta^c = \eta$, or in other words,

$$\eta = C(\bar{\eta})^T \text{ and } \bar{\eta} = -(\eta)^T C^{-1}. \quad \text{A.16}$$

They can be constructed from chiral spinors as follows:

$$\eta = \psi_R + \psi_L^c, \quad \zeta = \psi_L + \psi_R^c. \quad \text{A.17}$$

As promised, we now give some Dirac matrix trace identities. These are taken verbatim out of Bjorken and Drell [Bjo.64], but first a definition:

$$\not{a} \equiv a_\mu \gamma^\mu. \quad \text{A.18}$$

Next we have some non-trace identities:

$$\begin{aligned} \gamma_\mu \gamma^\mu &= 4, \\ \gamma_\mu \not{a} \gamma^\mu &= -2\not{a}, \\ \gamma_\mu \not{a} \not{b} \gamma^\mu &= 4a \cdot b, \\ \gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu &= -2\not{c} \not{b} \not{a}. \end{aligned} \quad \text{A.19}$$

Finally, we have the trace identities:

$$\text{Tr} [\not{a}_1 \cdots \not{a}_n] = 0 \text{ for } n \text{ odd}. \quad \text{A.20}$$

$$\text{Tr} [\mathbb{1}] = 4, \quad \text{A.21}$$

$$\text{Tr} [\not{a} \not{b}] = 4a \cdot b, \quad \text{A.22}$$

$$\text{Tr} [\not{a}_1 \not{a}_2 \not{a}_3 \not{a}_4] = 4[a_1 \cdot a_2 a_3 \cdot a_4 + a_1 \cdot a_4 a_2 \cdot a_3 - a_1 \cdot a_3 a_2 \cdot a_4], \quad \text{A.23}$$

and

$$\text{Tr} [\not{a}_1 \cdots \not{a}_n] = a_1 \cdot a_2 \text{Tr} [\not{a}_3 \cdots \not{a}_n] - a_1 \cdot a_3 \text{Tr} [\not{a}_2 \not{a}_4 \cdots \not{a}_n] + \cdots + a_1 \cdot a_n \text{Tr} [\not{a}_2 \cdots \not{a}_{n-1}]. \quad \text{A.24}$$

$$\text{Tr} [\gamma_5] = 0, \quad \text{A.25}$$

$$\text{Tr} [\gamma_5 \not{a} \not{b}] = 0, \quad \text{A.26}$$

and

$$\text{Tr} [\gamma_5 \not{a} \not{b} \not{c} \not{d}] = 4i\epsilon_{\alpha\beta\gamma\delta} a^\alpha b^\beta c^\gamma d^\delta. \quad \text{A.27}$$

APPENDIX B - SUSY FEYNMAN RULES

B.1 Gauge - Yukawa Couplings

The general gauge invariant SUSY Lagrangian written down in the first chapter is:

[DeW.75]

$$\mathcal{L} = \mathcal{L}_{\text{GAUGE}} + \mathcal{L}_{\text{MATTER}} + \mathcal{L}_{\text{SCALAR}} + \mathcal{L}_{\text{YUKAWA}}, \quad \text{B.1}$$

where

$$\mathcal{L}_{\text{GAUGE}} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{2}\bar{\chi}^a \gamma^\mu (D_\mu \chi)^a, \quad \text{B.1a}$$

$$\mathcal{L}_{\text{MATTER}} = (D_\mu \phi)_i^* (D_\mu \phi)^i + \bar{\psi}_L i \gamma^\mu D_\mu \psi_L^i + i\sqrt{2}g\bar{\chi}^a \phi_i^* T_j^{ai} \psi_L^j + i\sqrt{2}g\bar{\psi}_L i T_j^{ai} \phi^j \chi^a, \quad \text{B.2b}$$

$$\begin{aligned} \mathcal{L}_{\text{YUKAWA}} &= \frac{1}{2} \sum_{ij} \left[\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j} (\psi_L^j)^T C^{-1} \psi_L^i + \left(\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j} \right)^* \bar{\psi}_L^i C (\bar{\psi}_L^j)^T \right] \\ &= -\frac{1}{2} \sum_{ij} \left[\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j} \bar{\psi}_R^{ci} \psi_L^j - \left(\frac{\partial^2 W}{\partial \Phi^i \partial \Phi^j} \right)^* \bar{\psi}_L^j \bar{\psi}_R^{ci} \right], \end{aligned} \quad \text{B.1c}$$

$$\mathcal{L}_{\text{SCALAR}} = -\sum_i \left(\frac{\partial W}{\partial \Phi^i} \right)^* \left(\frac{\partial W}{\partial \Phi^i} \right) - \frac{1}{2} \sum_a (g\phi_i^* T_j^{ai} \phi^j - \xi^a)^2, \quad \text{B.1d}$$

where ξ^a appears only when $f_{abc}\xi^a = 0$, for all b and c (ie. one has an invariant abelian subalgebra), and

$$W = c_i \Phi^i + m_{ij} \Phi^i \Phi^j + g_{ijk} \Phi^i \Phi^j \Phi^k. \quad \text{B.2}$$

Finally the covariant derivatives are given by:

$$D_\mu \Phi = (\partial_\mu - igL \cdot B_\mu) \Phi, \quad \text{B.3a}$$

$$D_\mu \bar{\Phi} = (\partial_\mu + igL^T \cdot B_\mu) \bar{\Phi}, \quad \text{B.3b}$$

$$(D_\mu \chi)^a = \partial_\mu \chi^a + g f_{abc} B_\mu^b \chi^c, \quad \text{B.3c}$$

$$(D_\mu B_\nu)^a = -F_{\mu\nu}^a, \quad \text{B.3d}$$

$$(D_\mu D)^a = \partial_\mu D^a + g f_{abc} B_\mu^b D^c, \quad \text{B.3e}$$

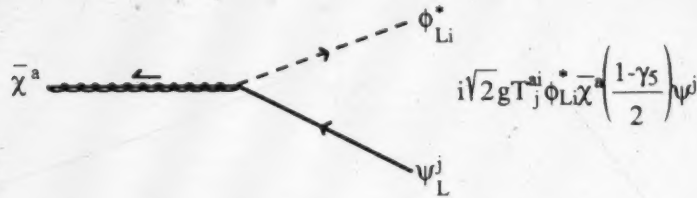
and

$$F_{\mu\nu}^a = \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + gf_{abc} B_\mu^b B_\nu^c. \quad B.4$$

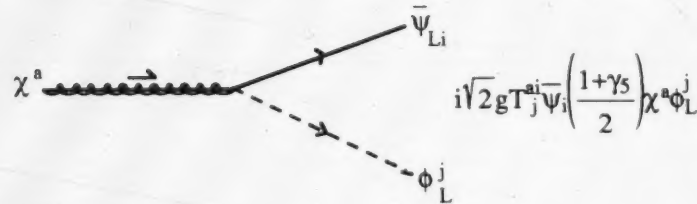
We wish to look at the third and fourth terms of $\mathcal{L}_{\text{MATTER}}$, to get the Feynman rules for "gauge Yukawa" coupling to gauge fermions, χ^a :

$$\begin{aligned} \mathcal{L}_{\text{G-Y}} &= i\sqrt{2}gT_j^{ai}\phi_L^*\bar{\chi}^a\psi_L^j + i\sqrt{2}gT_j^{ai}\bar{\psi}_L\chi^a\phi_L^j \\ &= i\sqrt{2}gT_j^{ai}\phi_L^*\bar{\chi}^a\left(\frac{1-\gamma_5}{2}\right)\psi_L^j + i\sqrt{2}gT_j^{ai}\bar{\psi}_L\left(\frac{1+\gamma_5}{2}\right)\chi^a\phi_L^j, \end{aligned} \quad B.5$$

where g is the gauge coupling. We now may obtain the vertex functions, since the vertex functions in this case are just given by the Lagrangian [Ram.89]:



$$i\sqrt{2}gT_j^{ai}\phi_L^*\bar{\chi}^a\left(\frac{1-\gamma_5}{2}\right)\psi_L^j, \quad B.6a$$



$$i\sqrt{2}gT_j^{ai}\bar{\psi}_L\left(\frac{1+\gamma_5}{2}\right)\chi^a\phi_L^j. \quad B.6b$$

B.2 Two Fermion - Two Scalar Couplings

In a non-renormalizable supersymmetric theory, the Yukawa portion of the Lagrangian may be written as [Wei.81]:

$$\begin{aligned} \mathcal{L} &= (\psi_L^i)^T C^{-1} \psi_L^j \left[\frac{\partial^2 f}{\partial \Phi^i \partial \Phi^j} - J_{km}^{-1} \left(\frac{\partial f}{\partial \Phi^k} \right) \frac{\partial^3 d}{\partial \Phi_m^* \partial \Phi^i \partial \Phi^j} \right] \\ &+ \bar{\psi}_L C (\bar{\psi}_L)^T \left[\left(\frac{\partial^2 f}{\partial \Phi^i \partial \Phi^j} \right)^* - J_{mk}^{-1} \left(\frac{\partial f}{\partial \Phi^k} \right)^* \frac{\partial^3 d}{\partial \Phi^m \partial \Phi_i^* \partial \Phi_j^*} \right], \end{aligned} \quad B.6$$

where $f(\Phi)$ is a function of chiral supermultiplets and is the equivalent of the

superpotential, W , and $d(\Phi, \Phi^*)$ is a function of the chiral supermultiplets and their complex conjugates, and

$$J_{ij} = \frac{\partial^2 d}{\partial \Phi^i \partial \Phi^j}. \quad \text{B.7}$$

The Lagrangian for a two fermion - two scalar, f term is then:

$$\mathcal{L} = \frac{g}{M_c} (\bar{\psi}_R^1 \psi_L^2 \phi^3 \phi^4 - \bar{\psi}_L^2 \psi_R^1 \phi_3^* \phi_4^*), \quad \text{B.8}$$

corresponding to a vertex function:

$$\frac{g}{M_c} \bar{\psi}_L^1 \psi_R^2 \phi^3 \phi^4, \quad \text{B.9a}$$

$$- \frac{g}{M_c} \bar{\psi}_R^2 \psi_L^1 \phi_3^* \phi_4^*. \quad \text{B.9b}$$

APPENDIX C - SUPERMULTIPLY QUANTUM GAUGE NUMBERS

Chiral Left Supermultiplet	SU(3)	SU(2)	Y	R-parity $(-1)^{3B+2S+L}$	
				spin 1/2	spin 0
$Q_L = (U_L, D_L)$	3	2	-1/6	1	-1
U_L^c	$\bar{3}$	1	2/3	1	-1
D_L^c	$\bar{3}$	1	-1/3	1	-1
$L = (N_L, E_L)$	1	2	1/2	1	-1
E_L^c	1	1	-1	1	-1
$H_L = (H_L^0, H_L^-)$	1	2	1/2	-1	1
$H_L' = (H_L'^+, H_L'^0)$	1	2	-1/2	-1	1

Figure C.1: The Quantum Gauge Numbers of the Chiral Left Supermultiplets of Supersymmetry. An R-parity of 1 would denote Standard Model particles (except for the extra Higgs).

APPENDIX D - INTEGRALS

In this thesis we will require many integrations over 4-space. This appendix will seek to demonstrate how these integrals can be performed and then shall catalogue the necessary ones.

We will start out with a simple identity; the Feynman parametrization:

$$\frac{1}{a_1 a_2 \cdots a_n} = (n-1)! \int_0^1 \frac{dz_1 dz_2 \cdots dz_n}{(a_1 z_1 + a_2 z_2 + \cdots + a_n z_n)^n} \delta(1 - \sum_{i=1}^n z_i). \quad (D.1)$$

One can fiddle with this to expand its use, such as differentiating it with respect to a_1 ,

$$\frac{1}{a_1^2 a_2 \cdots a_n} = n! \int_0^1 \frac{z_1 dz_1 dz_2 \cdots dz_n}{(a_1 z_1 + a_2 z_2 + \cdots + a_n z_n)^{n+1}} \delta(1 - \sum_{i=1}^n z_i). \quad (D.2)$$

Continuing on in this manner leads to the most general form:

$$\frac{1}{\prod_{i=1}^n a_i^{\alpha_i}} = \frac{\Gamma(\alpha)}{\prod_{i=1}^n \Gamma(\alpha_i)} \int_0^1 \prod_{i=1}^n z_i^{\alpha_i-1} dz_i \frac{\delta(1-z)}{\left(\sum_{i=1}^n a_i z_i\right)^\alpha}, \quad (D.3)$$

with

$$\alpha = \sum_{i=1}^n \alpha_i, \quad (D.4)$$

and

$$z = \sum_{i=1}^n z_i. \quad (D.5)$$

We now proceed onto the major portion of this appendix, namely the evaluation of d^4k integrals. These will be evaluated by first Euclideanizing them and then using 4-dimensional spherical integration. What we wish to evaluate is the general integral

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{k^{2m}}{(k^2 - m^2)^n}, \quad (D.6)$$

or more explicitly,

$$I = \frac{1}{(2\pi)^4} \int d^3k \int_{-\infty}^{+\infty} dk_0 \frac{(k_0^2 - \vec{k}^2)^m}{[(k_0 - \sqrt{\vec{k}^2 - m^2} + i\epsilon)(k_0 + \sqrt{\vec{k}^2 - m^2} - i\epsilon)]^n}. \quad (D.7)$$

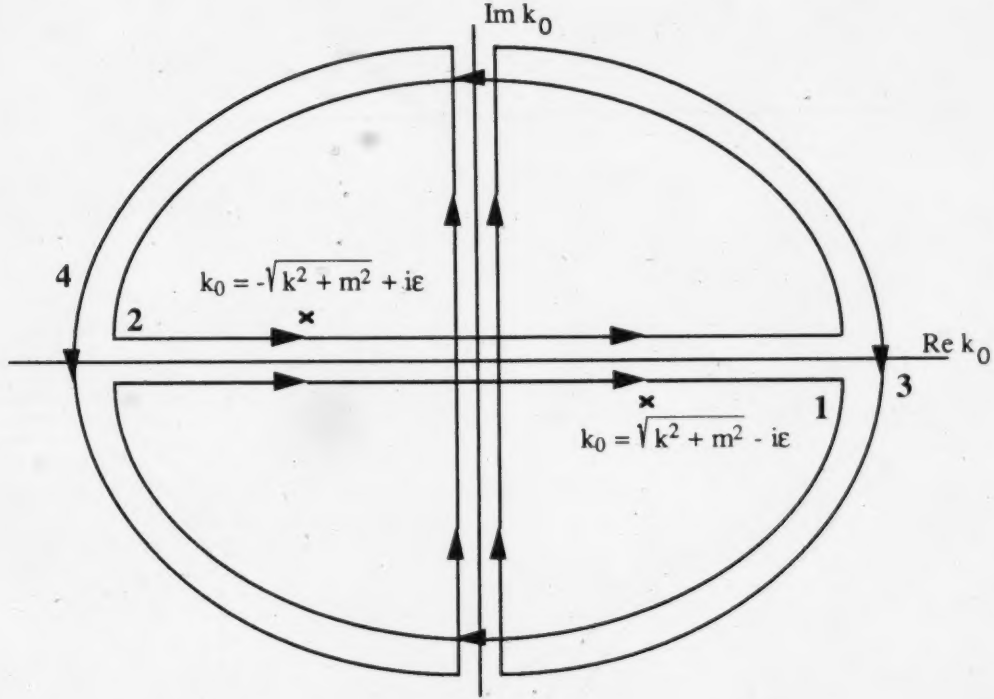


Figure D.1: Contour integrations for the d^4k integrations. Contour 1 can be rotated to contour 3, while contour 2 can be rotated to contour 4.

We can now change the limits of integration on k_0 to $(-\infty, i\infty)$ simply by rotating the contour integration by 90° counterclockwise (see figure D.1) so that

$$I = \frac{1}{(2\pi)^4} \int d^3k \int_{-i\infty}^{+i\infty} dk_0 \frac{(k_0^2 - \vec{k}^2)^m}{[(k_0 - \sqrt{\vec{k}^2 - m^2} + i\epsilon)(k_0 + \sqrt{\vec{k}^2 - m^2} - i\epsilon)]^n}. \quad (D.8)$$

Making the substitution $k_0 \rightarrow ik_0$, one has

$$I = \frac{i}{(2\pi)^4} \int d^3k \int_{-\infty}^{+\infty} dk_0 \frac{(-1)^m (k_0^2 + \vec{k}^2)^m}{(-1)^n (k_0^2 + \vec{k}^2 + m^2)^n}, \quad (D.9)$$

or

$$I = \frac{i(-1)^{m-n}}{(2\pi)^4} \int d^4k \frac{k^{2m}}{(k^2 + m^2)^n}, \quad (D.10)$$

where now the k^2 is an Euclidean metric k^2 . This can now be evaluated using the fact that in n dimensions, a spherically symmetric function integrated over the n -sphere is given by:

$$\int f(r) d^n x = \frac{2\pi^{n/2}}{\Gamma(n/2)} \int f(r) r^{n-1} dr, \quad (D.11)$$

which for $n=4$ gives

$$\int f(r) d^4 x = 2\pi^2 \int_0^\infty f(r) r^3 dr. \quad (D.12)$$

In its final form our integral is now

$$I = \frac{i(-1)^{m-n}}{8\pi^2} \int_0^\infty dk \frac{k^{2m+3}}{(k^2 + m^2)^n}. \quad (D.13)$$

Let's now apply this to the specific integral

$$\begin{aligned} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^3} &= \frac{-i}{8\pi^2} \int_0^\infty \frac{k^3 dk}{(k^2 + m^2)^3} = \frac{-i}{16\pi^2} \int_0^\infty \frac{x dx}{(x + m^2)^3} \\ &= \frac{-i}{16\pi^2} \int_{m^2}^\infty \frac{(y - m^2) dy}{y^3} = \frac{-i}{16\pi^2} \left[\frac{-1}{y} + \frac{m^2}{2y^2} \right]_{m^2}^\infty = \frac{-i}{16\pi^2 m^2} \left(1 - \frac{1}{2} \right) \\ &= \frac{-i}{32\pi^2 m^2}. \end{aligned} \quad (D.14)$$

It is also desirable to calculate the above integral when not all the masses in the denominator are not the same. Thus we calculate

$$\begin{aligned}
& \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^2 (k^2 - \alpha m^2)} = \frac{-i}{16\pi^2 m^2} \int_0^\infty \frac{x dx}{(x+1)^2 (x+\alpha)} \\
&= \frac{-i}{16\pi^2 m^2} \int_1^\infty \frac{(y-1) dx}{y^2 (y+\alpha-1)} = \frac{-i}{16\pi^2 m^2} \int_1^\infty \frac{(y-1) dy}{(\alpha-1)^2} \left[\frac{(\alpha-1)-y}{y^2} + \frac{1}{(y+\alpha-1)} \right] \\
&= \frac{-i}{16\pi^2 m^2} \left[\frac{(\alpha-1) \left[\ln y + \frac{1}{y} \right] - y + \ln y + y + \alpha - 1 - \alpha \ln (y+\alpha-1)}{(\alpha-1)^2} \right]_1^\infty \quad (D.15) \\
&= \frac{-i}{16\pi^2 m^2} \frac{1}{(\alpha-1)^2} \left[\alpha \ln y - \alpha \ln (y+\alpha-1) + \frac{(\alpha-1)}{y} + \alpha - 1 \right]_1^\infty \\
&= \frac{-i}{16\pi^2 m^2} \frac{\alpha \ln \alpha - \alpha - 1}{(\alpha-1)^2}.
\end{aligned}$$

In a similar fashion one can find that

$$\begin{aligned}
& \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{(k^2 - m^2)^4 (k^2 - \alpha m^2)} \\
&= \frac{i}{192\pi^2 m^4} \frac{(\alpha-1)(4\alpha^2 + 10\alpha - 2) - 12\alpha^2 \ln \alpha}{(\alpha-1)^4} \quad (D.16) \\
&\rightarrow \frac{i}{192\pi^2 m^4} \text{ for } \alpha \rightarrow 1.
\end{aligned}$$

Other integrals are:

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^4} = \frac{i}{96\pi^2 m^4}, \quad (D.17)$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^5} = \frac{-i}{192\pi^2 m^4}, \quad (D.18)$$

and

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k^4}{(k^2 - m^2)^5} = \frac{-i}{64\pi^2 m^4}. \quad (D.19)$$

More generally, one has that

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k^{2n}}{\prod_{i=1}^m (k^2 - \alpha_i)} = \frac{-i}{16\pi^2} \sum_{i=1}^m \frac{\alpha_i^{n+1} \ln \alpha_i}{\prod_{j \neq i} (\alpha_i - \alpha_j)}. \quad (D.20)$$

As our last important integral, we have the integral

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{(k^2 - m^2)^3}. \quad (D.21)$$

This integral is logarithmically divergent, so some care must be taken to obtain the proper regularization. We will regularize it with a cutoff Λ . Evaluating the integral, one has:

$$\begin{aligned} \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{(k^2 - M^2)^3} &= \frac{i}{8\pi^2} \int_0^\Lambda \frac{k^5 dk}{(k^2 - M^2)^3} = \frac{i}{16\pi^2} \int_0^\Lambda \frac{x^2 dx}{(x - M^2)^3} \\ &= \frac{i}{16\pi^2} \int_{M^2}^{\Lambda^2 + M^2} \frac{(y^2 - 2M^2 y + M^4) dy}{y^3} = \frac{i}{16\pi^2} \left[\ln y + \frac{2M^2}{y} - \frac{M^4}{2y^2} \right]_{M^2}^{\Lambda^2 + M^2} \quad (D.22) \\ &= \frac{i}{16\pi^2} \left[\ln \left(\frac{\Lambda^2 + M^2}{M^2} \right) - 2 - \frac{1}{2} \right] = \frac{i}{16\pi^2} \ln \frac{M^2}{\Lambda^2}, \end{aligned}$$

where the last approximation is only valid since the numerical constants that were dropped have no M dependence and therefore will disappear when this integral is summed over internal quarks.

APPENDIX E - FIERZ IDENTITIES

It is sometimes necessary or convenient in a four-fermion interaction to switch which spinors are contracted with the adjoint spinors. Fortunately this is possible due to the fact that any direct product can be decomposed into a sum of other direct products [Bai.77]

$$A_{ij}B_{km} = \sum_r C_{im}^{(r)} D_{kj}^{(r)}. \quad E.1$$

In practice A and B are particular combinations of the gamma matrices. If A and B are both the 4x4 identity matrix, $\mathbb{1}$, then its behavior under the Lorentz transformation, plus the requirement that the product transforms as a scalar, restricts the possibilities to

$$(\mathbb{1})_{ij}(\mathbb{1})_{km} = a_S(\mathbb{1})_{im}(\mathbb{1})_{kj} + a_P(\gamma_5)_{im}(\gamma_5)_{kj} + a_V(\gamma^\mu)_{im}(\gamma_\mu)_{kj} + a_A(\gamma^\mu\gamma_5)_{im}(\gamma_\mu\gamma_5)_{kj} + a_T(\sigma^{\mu\nu})_{im}(\sigma_{\mu\nu})_{kj}. \quad E.2$$

Multiplying this last equation by $(\mathbb{1})_{jk}$, $(\gamma_5)_{jk}$, $(\gamma_\mu)_{jk}$, $(\gamma_\mu\gamma_5)_{jk}$, and $(\sigma_{\mu\nu})_{jk}$ respectively, and using the trace identities to be found at the end of Appendix A, one finds that

$$a_S = a_P = a_V = -a_A = 2a_T = \frac{1}{4}. \quad E.3$$

In an obvious shorthand, therefore, we have

$$\mathbb{1} \otimes \mathbb{1} = \frac{1}{4} \left\{ \mathbb{1} \otimes \mathbb{1} + \gamma_5 \otimes \gamma_5 + \gamma^\mu \otimes \gamma_\mu - \gamma^\mu \gamma_5 \otimes \gamma_\mu \gamma_5 + \frac{1}{2} \sigma^{\mu\nu} \otimes \sigma_{\mu\nu} \right\}, \quad E.4$$

and more generally

$$A \otimes B = \frac{1}{4} \left\{ A \mathbb{1} B \otimes \mathbb{1} + A \gamma_5 B \otimes \gamma_5 + A \gamma^\mu B \otimes \gamma_\mu - A \gamma^\mu \gamma_5 B \otimes \gamma_\mu \gamma_5 + \frac{1}{2} A \sigma^{\mu\nu} B \otimes \sigma_{\mu\nu} \right\}. \quad E.5$$

Using (E.5), we obtain the following Fierz identities, this time adding an extra minus sign to account for the anticommutation of the associated spinors:

$$R \otimes R = (-) \frac{1}{2} \left[R \otimes R + \frac{1}{4} R \sigma_{\mu\nu} \otimes \sigma^{\mu\nu} \right], \quad E.6$$

$$L \otimes L = (-) \frac{1}{2} \left[L \otimes L + \frac{1}{4} L \sigma_{\mu\nu} \otimes \sigma^{\mu\nu} \right], \quad E.7$$

$$R \otimes L = (-) \frac{1}{2} \gamma_\mu L \otimes \gamma^\mu R, \quad E.8$$

$$L \otimes R = (-) \frac{1}{2} \gamma_\mu R \otimes \gamma^\mu L. \quad E.9$$

APPENDIX F - SOME FUN WITH SU(3)

In this thesis we will be interested in the summed products of various combinations of the fundamental and adjoint representations of SU(3). Let's start out with a simple trace theorem for the fundamental or $\underline{3}$ representation, namely

$$\text{Tr}(T_a T_b) = \frac{1}{2} \delta_{ab}. \quad (\text{F.1})$$

This can be generalized to any representation [Geo.82]

$$\text{Tr}(X_a X_b) = k_D \delta_{ab}, \quad (\text{F.2})$$

where obviously $k_D = 1/2$ for the $\underline{3}$ of SU(3). Unknown k_D 's can be found using

$$k_{D_1 \oplus D_2} = k_{D_1} + k_{D_2}, \quad (\text{F.3a})$$

$$k_{D_1 \otimes D_2} = \dim(D_1) \cdot k_{D_1} + \dim(D_2) \cdot k_{D_2}. \quad (\text{F.3b})$$

For the $\underline{8}$ of SU(3), where $\underline{3} \otimes \underline{\bar{3}} = \underline{8} \oplus \underline{1}$, with $k_1 = 0$, one would have $k_8 = (3 \cdot \frac{1}{2}) \times 2 = 3$.

The structure functions, f_a , define the adjoint representation of SU(3). These structure functions are defined by:

$$[X_a, X_b] = i f_{abc} X_c. \quad (\text{F.4})$$

By explicit calculation, $[f_a, f_b] = -f_{abc} f_c$, or

$$\left[\frac{f_a}{i}, \frac{f_b}{i} \right] = i f_{abc} \frac{f_c}{i}, \quad (\text{F.5})$$

suggesting f_a/i is what should be identified as the adjoint representation. Since the adjoint representation of SU(3) is the $\underline{8}$, one expects that

$$\text{Tr} \left(\frac{f_a}{i} \frac{f_b}{i} \right) = 3 \delta_{ab} \quad \text{or} \quad \text{Tr}(f_a f_b) = -3 \delta_{ab}. \quad (\text{F.6})$$

Moving onto some of the summed products we wish to find, the first of which is $\sum_a T_a T_a$. By completeness, we know this must be some multiple of the unit matrix. Since

$$\sum_a \text{Tr}(T_a T_a) = 4, \quad (\text{F.7})$$

and $\text{Tr}(\mathbf{1}) = 3$, we have that

$$\sum_a T_a T_a = \frac{4}{3} \mathbf{1}. \quad (\text{F.8})$$

Next we have

$$\begin{aligned}
& \sum_{a,c} f_{abc} T_a T_c \\
&= \frac{1}{2} f_{abc} [(T_a T_c + T_c T_a) + (T_a T_c - T_c T_a)] \\
&= \frac{1}{2} f_{abc} [T_a, T_c] \text{ since only the antisymmetric part contributes} \quad (F.9) \\
&= \frac{1}{2} f_{abc} f_{acd} T_d = \frac{1}{2} \text{Tr}(f_b f_d) T_d \\
&= -\frac{3}{2} i T_b.
\end{aligned}$$

In the above, and in what follows, $[a,b]$ represents the commutator and $\{a,b\}$ represents the anticommutator of a and b . Finally, we have

$$\begin{aligned}
& \sum_a T_a T_b T_a \\
&= (T_b T_a + i f_{abc} T_c) T_a \\
&= \frac{4}{3} T_b + i \cdot \frac{3}{2} i T_b \\
&= -\frac{1}{6} T_b.
\end{aligned} \quad (F.10)$$

We will also require some trace theorems for three and four adjoint representations. For three, one has

$$\begin{aligned}
& \text{Tr}(f_a f_b f_c) \\
&= \frac{1}{2} \text{Tr}[[f_a, f_b] f_c + \{f_a, f_b\} f_c] \\
&= \frac{1}{2} \text{Tr}[[f_a, f_b] f_c] \\
&= -\frac{1}{2} f_{abd} \text{Tr}[f_d f_c] \\
&= \frac{3}{2} f_{abc}.
\end{aligned} \quad (F.11)$$

While for four, one has

$$\begin{aligned}
& \text{Tr}(f_a f_b f_c f_d) \\
&= \frac{1}{4} \text{Tr}[([f_a, f_b] + \{f_a, f_b\}) \times ([f_c, f_d] + \{f_c, f_d\})] \\
&= \frac{1}{2} \text{Tr}[[f_a, f_b] [f_c, f_d] + \{f_a, f_b\} \{f_c, f_d\}] \\
&= \frac{1}{4} f_{abe} f_{cdf} \text{Tr}[f_e f_f] + \text{symmetric part} \\
&= -\frac{3}{4} f_{abe} f_{cde} + \text{symmetric part}.
\end{aligned} \quad (F.12)$$

APPENDIX G - CALCULATIONS RELATIVE TO NEDM

In order to simplify equation (3.22) we must deal with two traces, the SU(3) adjoint representation trace $\text{Tr} [f_a f_b f_c f_d f_e]$, (with $(f_a)_{bc} = f_{abc}$) and the Dirac matrix trace,

$$\text{Tr} \left[\sigma^{\lambda} \gamma^{\mu} \frac{1}{k-m} \gamma^{\nu} \frac{1}{k-m} \gamma^{\rho} \frac{1}{k-m} \gamma^{\delta} \frac{1}{k-m} \gamma^{\rho} \frac{1}{k-m} \right]. \quad (\text{G.1})$$

This last trace is then integrated over $d^4 k / (2\pi)^4$, so results odd in k can be ignored. Furthermore, we do not wish to have results symmetric in any of (μ, ν, δ, ρ) . To see this, suppose it was symmetric in μ, ν , then the resulting Dirac trace would be antisymmetric in δ, ρ . This means the adjoint representation indices would be symmetric in b, c and antisymmetric in indices d, e resulting in a adjoint trace solution that would involve f_{def} but not f_{abg} . This would not be of a form that would lead to the Weinberg's operator. As a result we can ignore terms that involve k^{μ} 's, as symmetric integration takes terms like $k^{\mu} k^{\nu}$ to $g^{\mu\nu}$. Equation (G.1) can then be simplified to

$$\frac{(m^5 - 2m^3 k^2 + m k^4)}{(k^2 - m^2)^5} \text{Tr} [\sigma^{\lambda} \gamma^{\mu} \gamma^{\nu} \gamma^{\delta} \gamma^{\rho}]. \quad (\text{G.2})$$

The $d^4 k$ integration can now be easily performed (Appendix D) and Reduce2 can be used to evaluate the Dirac trace:

$$\begin{aligned} & \text{Tr} [\sigma^{\lambda} \gamma^{\mu} \gamma^{\nu} \gamma^{\delta} \gamma^{\rho}] \\ &= 4 \{ -g_{\delta\sigma} \epsilon_{\lambda\rho\nu\mu} + g_{\rho\sigma} \epsilon_{\lambda\delta\nu\mu} + g_{\nu\sigma} \epsilon_{\lambda\rho\delta\mu} - g_{\mu\sigma} \epsilon_{\lambda\rho\delta\nu} - g_{\lambda\delta} \epsilon_{\rho\nu\mu\sigma} \\ & \quad + g_{\lambda\rho} \epsilon_{\delta\nu\mu\sigma} + g_{\rho\nu} \epsilon_{\lambda\delta\mu\sigma} - g_{\mu\nu} \epsilon_{\lambda\rho\delta\sigma} - g_{\lambda\mu} \epsilon_{\rho\delta\nu\sigma} + g_{\lambda\nu} \epsilon_{\rho\delta\mu\sigma} \\ & \quad + g_{\delta\mu} \epsilon_{\lambda\rho\nu\sigma} - g_{\rho\mu} \epsilon_{\lambda\delta\nu\sigma} - g_{\delta\nu} \epsilon_{\lambda\rho\mu\sigma} + g_{\rho\delta} \epsilon_{\lambda\nu\mu\sigma} \} \\ &= 4 \{ -g_{\delta\sigma} \epsilon_{\lambda\rho\nu\mu} + g_{\rho\sigma} \epsilon_{\lambda\delta\nu\mu} + g_{\nu\sigma} \epsilon_{\lambda\rho\delta\mu} - g_{\mu\sigma} \epsilon_{\lambda\rho\delta\nu} \\ & \quad - g_{\lambda\delta} \epsilon_{\rho\nu\mu\sigma} + g_{\lambda\rho} \epsilon_{\delta\nu\mu\sigma} - g_{\lambda\mu} \epsilon_{\rho\delta\nu\sigma} + g_{\lambda\nu} \epsilon_{\rho\delta\mu\sigma} \}, \end{aligned} \quad (\text{G.3})$$

where once again we have ignored terms symmetric in (μ, ν, δ, ρ) . When contracted with $G_{\sigma\lambda}^a A_{\mu}^b A_{\nu}^c A_{\delta}^d A_{\rho}^e$ and multiplied by $\text{Tr} [f_a f_b f_c f_d f_e]$, as in equation (3.22), then our above trace, with a rearrangement of dummy indices, is just $-32 g_{\delta\sigma} \epsilon_{\lambda\rho\nu\mu}$. We now perform the adjoint representation trace realizing that it must be antisymmetric in b, c . Thus we have

$$\text{Tr} [f_a [f_b, f_c] f_d f_e] = -f_{bcf} \text{Tr} [f_a f_f f_d f_e] = \frac{-3}{4} f_{bcf} f_a f_g f_{deg} + \text{parts sym. in } \{a, f\}, \{d, e\}. \quad (\text{G.4})$$

APPENDIX H - SOME CALCULATIONS FOR $b \rightarrow s\gamma^*$

This appendix will deal with the simplification process that leads from equation (4.17) to equation (4.19). We start with the evaluation of the vertex function for the appropriate 4-momentum values:

$$\begin{aligned} & \Gamma_{\mu\alpha\beta}(q, -k-qy+p_2x, k+q(y-1)-p_2x) \\ &= \frac{-ie\lambda_\gamma}{M_W^2} \left\{ -(k+qy-p_2x) \cdot (k+q(y-1)-p_2x) (q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) \right. \\ & \quad + (k+q(y-1)-p_2x)_\alpha [(k+qy-p_2x)_\mu q_\beta - (k+qy-p_2x) \cdot q g_{\beta\mu}] \\ & \quad - (k+qy-p_2x)_\beta [(k+q(y-1)-p_2x)_\mu q_\alpha - (k+q(y-1)-p_2x) \cdot q g_{\alpha\mu}] \\ & \quad \left. + g_{\alpha\beta} [(k+qy-p_2x) \cdot q (k+q(y-1)-p_2x)_\mu - (k+q(y-1)-p_2x) \cdot q (k+qy-p_2x)_\mu] \right\}. \end{aligned} \quad (H.1)$$

This will be multiplied by $(k + qy + p_2(1-x))_\nu$, however, only the k^2 terms will be kept with $k_\mu k_\nu \rightarrow \frac{1}{4}k^2 g_{\mu\nu}$. This results in:

$$\begin{aligned} & \frac{-ie\lambda_\gamma k^2}{M_W^2} \left\{ -(qy+p_2(1-x))_\nu (q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) - \frac{1}{4}(qy-p_2x+q(y-1)-p_2x)_\nu (q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) \right. \\ & \quad + \frac{1}{4}(qy+p_2(1-x))_\nu (q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) + \frac{1}{4}g_{\alpha\nu} [(qy-p_2x)_\mu q_\beta - (qy-p_2x) \cdot q g_{\beta\mu}] \\ & \quad + \frac{1}{4}(q(y-1)-p_2x)_\alpha (q_\beta g_{\mu\nu} - q_\nu g_{\beta\mu}) - \frac{1}{4}(qy+p_2(1-x))_\nu (q_\alpha g_{\beta\mu} - q_\beta g_{\alpha\mu}) \\ & \quad - \frac{1}{4}g_{\beta\nu} [(q(y-1)-p_2x)_\mu q_\alpha - (q(y-1)-p_2x) \cdot q g_{\alpha\mu}] - \frac{1}{4}(qy-p_2x)_\beta (q_\alpha g_{\mu\nu} - q_\nu g_{\alpha\mu}) \\ & \quad \left. + \frac{1}{4}g_{\alpha\beta} [q_\nu (q(y-1)-p_2x)_\mu - (q(y-1)-p_2x) \cdot q g_{\mu\nu} - (qy-p_2x) \cdot q g_{\mu\nu} - q_\nu (qy-p_2x)_\mu] \right\} \end{aligned} \quad (H.2)$$

$$\begin{aligned} &= \frac{-ie\lambda_\gamma k^2}{4M_W^2} \left\{ -((q(1-4y)+p_2(4x-2))_\nu (q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) - g_{\alpha\beta} [q_\nu q_\mu - q^2 g_{\mu\nu}] \right. \\ & \quad + g_{\alpha\nu} [(qy-p_2x)_\mu q_\beta - (qy-p_2x) \cdot q g_{\beta\mu}] \\ & \quad - g_{\beta\nu} [(q(y-1)-p_2x)_\mu q_\alpha - (q(y-1)-p_2x) \cdot q g_{\alpha\mu}] \\ & \quad \left. + (q(y-1)-p_2x)_\alpha (q_\beta g_{\mu\nu} - q_\nu g_{\beta\mu}) - (qy-p_2x)_\beta (q_\alpha g_{\mu\nu} - q_\nu g_{\alpha\mu}) \right\}. \end{aligned}$$

Contracting with $\gamma^\alpha \gamma^\nu \gamma^\beta$, one has

$$\begin{aligned}
& \frac{-ie\lambda_\gamma k^2}{4M_W^2} \left\{ 4(\not{q}\not{q}_\mu - q^2\gamma_\mu) + (4x-2)(\gamma_\mu \not{p}_2 \not{q} - \not{q} \not{p}_2 \gamma_\mu) \right. \\
& \quad \left. + x(\not{p}_2 \not{q} \gamma_\mu - \not{p}_2 \gamma_\mu \not{q} + \not{q} \gamma_\mu \not{p}_2 - \gamma_\mu \not{q} \not{p}_2) \right\} \\
& = \frac{-ie\lambda_\gamma k^2}{M_W^2} \left\{ (\not{q}\not{q}_\mu - q^2\gamma_\mu) + (3x-1)(\not{p}_2 \not{p}_{1\mu} + \not{p}_1 \not{p}_{2\mu} - \not{p}_2 \gamma_\mu \not{p}_1 - \not{p}_1 \gamma_\mu \not{p}_2) \right\},
\end{aligned} \tag{H.3}$$

where one has used $\not{q}\gamma_\mu \not{q} = 2\not{q}\not{q}_\mu - q^2\gamma_\mu$, and for combinations of q and p_2 , one has broken q into $q = p_1 - p_2$. Finally using

$$\begin{aligned}
& \langle \bar{s}_L | \not{p}_2 \not{p}_{1\mu} + \not{p}_1 \not{p}_{2\mu} - \not{p}_2 \gamma_\mu \not{p}_1 - \not{p}_1 \gamma_\mu \not{p}_2 | b_L \rangle \\
& = \langle \bar{s} | \frac{1}{2} (m_b R + m_s L) \sigma_{\mu\nu} q^\nu - \frac{1}{2} (\not{q}\not{q}_\mu - q^2\gamma_\mu) L | b \rangle,
\end{aligned} \tag{H.4}$$

one has:

$$\frac{-ie\lambda_\gamma k^2}{2M_W^2} \left\{ (3-3x)(\not{q}\not{q}_\mu - q^2\gamma_\mu) + i(3x-1)(m_b R + m_s L) \sigma_{\mu\nu} q^\nu \right\}. \tag{H.5}$$

This gives us the final form found in equation (4.19).